Manonmaniam Sundaranar University,

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Tirunelveli - 627 012 Tamilnadu, India

OPEN AND DISTANCE LEARNING (ODL) PROGRAMMES

(FOR THOSE WHO JOINED THE PROGRAMMES FROM THE ACADEMIC YEAR 2023–2024)

B.Sc. Physics

III Year

Solid State Physics

Course Material

Prepared

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Tirunelveli - 12

Unit I

BONDING IN SOLIDS, CRYSTALSTRUCTURE : types of bonding–ionic bonding – bond energy of NaCl molecule –covalent bonding —Van-der-Waals bonding–crystal lattice–lattice translational vectors–lattice with basis –unit cell –Bravais lattices–crystal structure –packing of SCC, BCC, and FCC structures structures of NaCl and diamond crystals –Miller indices –procedure for finding them.

Unit II

ELEMENTARY LATTICE DYNAMICS: lattice vibrations and phonons: linear mono atomic and diatomic chains. Acoustical and optical phonons – Dulong and Petit‘s Law–properties of metals classical free Electron theory of metals (Drude-Lorentz) – Ohm‘s law–electrical and thermal conductivities – Weidemann - Franz‘ law.

Unit III

MAGNETIC PROPERTIES OF SOLIDS : permeability, susceptibility, relation between them –classification of magnetic materials –properties of dia, para, ferro, ferri and antiferromagnetism – Langevin‘s theory of diamagnetism –Weiss theory of Para magnetism–Curie-Weiss law –Weiss theory of ferromagnetism (qualitative only) – domains –B-H curve hysteresis and energy loss –soft and hard magnets.

Unit IV

DIELECTRIC PROPERTIES OF MATERIALS: Basic definitions polarization and electric susceptibility –local electric field of an atom dielectric constant and polarisability –polarization processes: electronic polarization–calculation of polarisability–ionic,orientational and space charge polarization –internal field –Clausius-Mosotti relation -frequency dependence of dielectric constant –dielectric loss – effect of temperature on dielectric constant.

Unit V

FERROELECTRIC & SUPERCONDUCTING PROPERTIES OF MATERIALS : Ferroelectriceffect : Curie-Weiss Law - ferroelectric domains, –elementary band theory : band gap (no derivation) - Hall effect –measurement of conductivity (four probe method) -Hall coefficient. Superconductivity: general properties of superconducting materials critical temperature –critical magnetic field – Meissner effect –isotope effect–type-I and type-II superconductors –London‘s equation and penetration depth.

TEXT BOOKS

1.Introduction to Solid State Physics, Kittel, Willey Eastern Ltd (2003).

2. Solid state Physics, Rita John,1stedition, Tata Mc Graw Hill publishers (2014).

3. Solid State Physics, RL Singhal, Kedarnath Ram Nath & Co., Meerut (2003)

4. Elements of Solid-State Physics, J.P. Srivastava,2ndEdition,2006, Prentice-Hall of India

5. Introduction to Solids, Leonid V. Azaroff , 2004, Tata Mc-Graw Hill

6. Solid State Physics, N.W. Ashcroft and N.D. Mermin,1976, Cengage Learning

7. Solid-state Physics, H. Ibach and H. Luth,2009, Springer

8. Elementary Solid-State Physics, 1/e M. Ali Omar, 1999, Pearson India

9. Solid-state Physics, M.A. Wahab, 2011, Narosa Publishing House, ND

Unit 1: Bondig Solids, Crystal Structure

1. Introduction
2. Ionic Bonding
3. Bond energy of NaCl Molecule
4. Covalent Bonding
5. Van-der Waals Bonding
6. Lattice Translational Vectors
7. Unit Cell
8. Bravais' Lattices
9. Crystal Structure packing of SCC, BCC and FCC
10. Structures of NaCal and diamond crystal
11. Procedure for finding Miller Indices

Introduction

Bonding in Solids

In solid-state physics, **bonding in solids** refers to the forces that hold atoms, ions, or molecules together to form a stable solid. These bonding forces determine how particles are arranged in space and directly influence the physical properties of solids such as hardness, melting point, electrical conductivity, and elasticity. Understanding bonding provides the foundation for explaining the structure and behavior of different types of solids.

Crystal Structure

The **crystal structure** of a solid describes the regular and repeating arrangement of its constituent particles in three-dimensional space. This orderly pattern, known as a crystal lattice, gives crystalline solids their characteristic shape and symmetry. The study of crystal structure helps in understanding the mechanical, thermal, and electrical properties of materials and plays a crucial role in solid-state physics and material science.

Ionic Bonding

Ionic bonding is one of the fundamental types of chemical bonding responsible for the formation of many solid compounds. It arises due to the **electrostatic force of attraction** between oppositely charged ions. This type of bonding is commonly observed in compounds formed between **metals and non-metals**, where there is a large difference in their electronegativities.

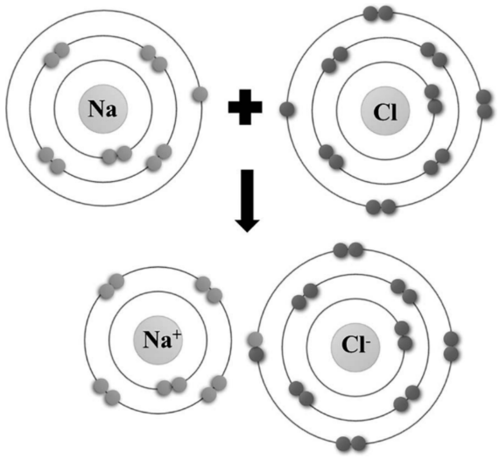


Figure 1. 1

Formation of Ionic Bond

Ionic bonding is formed by the **transfer of one or more electrons** from an atom of a metal to an atom of a non-metal.

- Metals tend to **lose electrons** and form **positive ions (cations)**.
- Non-metals tend to **gain electrons** and form **negative ions (anions)**.

This electron transfer leads to the formation of ions with stable electronic configurations (usually noble gas configuration).

Example: Sodium Chloride (NaCl)

- A sodium atom (Na) loses one electron to form Na^+.

- A chlorine atom (Cl) gains that electron to form Cl^-.
- The oppositely charged Na^+ and Cl^- ions attract each other, resulting in an ionic bond.

Nature of Ionic Bond

An ionic bond is **non-directional**, meaning the force of attraction acts equally in all directions around the ion. Unlike covalent bonds, ionic bonds do not involve sharing of electrons but rely purely on **electrostatic attraction**.

Ionic Crystal Lattice

In the solid state, ionic compounds do not exist as discrete molecules. Instead, they form a **three-dimensional crystal lattice**.

- Each positive ion is surrounded by negative ions.
- Each negative ion is surrounded by positive ions.
- This arrangement minimizes repulsion and maximizes attraction.

For example, in sodium chloride crystal:

- Each Na^+ ion is surrounded by six Cl^- ions.
- Each Cl^- ion is surrounded by six Na^+ ions.

This repeating arrangement gives ionic solids their characteristic **hard and brittle nature**.

Energy Changes in Ionic Bonding

The formation of an ionic bond involves:

1. **Ionization energy** (energy required to remove electrons from metal atoms),
2. **Electron affinity** (energy released when non-metal gains electrons),
3. **Lattice energy** (energy released when ions arrange themselves into a crystal lattice).

A high lattice energy makes the ionic solid more stable.

Characteristics of Ionic Compounds

- High melting and boiling points
- Hard and brittle in nature
- Soluble in polar solvents like water
- Conduct electricity in molten state or aqueous solution
- Exist as crystalline solids

Ionic bonding plays a vital role in determining the structure and properties of many solids. The strong electrostatic attraction between ions leads to the formation of stable crystalline structures, making ionic compounds essential in both physics and chemistry applications.

1.1 Bond energy of NaCl Molecule

The **bond energy of sodium chloride (NaCl)** refers to the energy associated with the formation and stability of the ionic bond between sodium (Na^+) and chloride (Cl^-) ions. Since NaCl is an **ionic compound**, its bond energy is best understood in terms of **lattice energy**, rather than a simple bond dissociation energy as in covalent molecules.

What is Bond Energy?

Bond energy is defined as the **amount of energy required to break a chemical bond** between atoms or ions.

In ionic compounds like NaCl, bond energy represents the **energy required to separate one mole of NaCl solid into free gaseous Na^+ and Cl^- ions**.

Formation of NaCl and Energy Changes

The formation of NaCl involves several energy steps:

1. **Ionization of Sodium Atom**

   Sodium atom loses one electron to form Na^+ ion.

   This process requires energy called **ionization energy**.

$Na \rightarrow Na^+ + e^-$ (Energy absorbed)

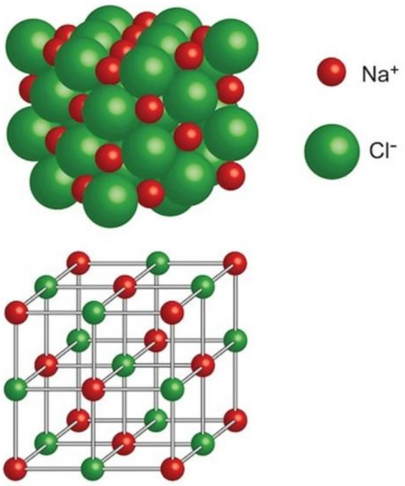


Figure. 1.2

2. **Electron Affinity of Chlorine Atom**

   Chlorine atom gains one electron to form Cl^- ion.

   This process releases energy known as **electron affinity**.

$Cl + e^- \rightarrow Cl^-$ (Energy released)

3. **Electrostatic Attraction between Ions**

   The oppositely charged Na^+ and Cl^- ions attract each other strongly due to Coulomb's force.

4. **Formation of Ionic Crystal Lattice**

   A large amount of energy is released when gaseous ions arrange themselves into a solid crystal lattice.

   This released energy is called **lattice energy**.

Born–Haber Cycle for NaCl

The **Born–Haber cycle** is used to explain and calculate the bond energy of NaCl by applying the law of conservation of energy.

It includes:

- Sublimation energy of sodium
- Ionization energy of sodium
- Dissociation energy of chlorine molecule
- Electron affinity of chlorine
- Lattice energy of NaCl

Among these, **lattice energy is the major contributor** to the high bond energy of NaCl.

Lattice Energy and Bond Energy

- Lattice energy is the **energy released when one mole of NaCl is formed from gaseous Na^+ and Cl^- ions**.
- For NaCl, lattice energy is **very high** due to:
    - High charge on ions
    - Small ionic radii
    - Strong electrostatic attraction

This high lattice energy makes NaCl:

- Hard
- Stable
- High melting point (~801°C)

Energy Curve Explanation

As Na^+ and Cl^- ions come closer:

- Attractive force increases
- Potential energy decreases
- At equilibrium distance, potential energy is minimum
  This minimum potential energy corresponds to the **bond energy of NaCl**.

1.2 Covalent Bonding

Atoms combine with one another to attain a stable electronic configuration, usually that of the nearest noble gas. When atoms are unable to lose or gain electrons completely, they achieve stability by **sharing electrons**. The chemical bond formed by such mutual sharing of electrons between atoms is known as **covalent bonding**. This type of bonding is commonly observed among **non-metallic elements**.

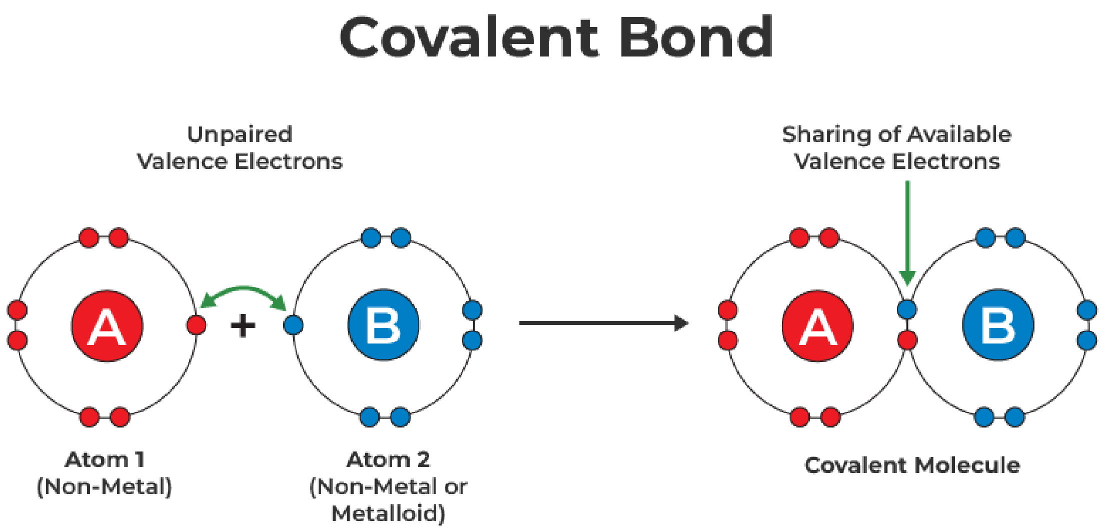


Figure 1. 2

A **covalent bond** is a chemical bond formed by the mutual sharing of one or more pairs of electrons between two atoms, so that each atom attains a stable outer electronic configuration.

Conditions for the Formation of Covalent Bond

Covalent bonding is favored under the following conditions:

1. The combining atoms are non-metals.
2. The difference in electronegativity between the atoms is small.
3. Neither atom can easily lose or gain electrons.
4. Both atoms require electrons to complete their outermost shell.

Formation of Covalent Bond

The formation of a covalent bond can be explained using the hydrogen molecule (H_2) as an example.

Each hydrogen atom has one electron in its valence shell and requires one more electron to attain a stable duplet configuration. Two hydrogen atoms share one pair of electrons. This shared pair of electrons is attracted by the nuclei of both atoms, resulting in the formation of a single covalent bond. Thus, both hydrogen atoms achieve stability.

Types of Covalent Bonds

Based on the number of electron pairs shared between atoms, covalent bonds are classified as follows:

1. Single Covalent Bond

When one pair of electrons is shared between two atoms, a single covalent bond is formed. Examples: H_2, Cl_2, CH_4

2. Double Covalent Bond

When two pairs of electrons are shared between two atoms, a double covalent bond is formed.

Examples: O_2, CO_2

3. Triple Covalent Bond

When three pairs of electrons are shared between two atoms, a triple covalent bond is formed.

Example: N_2

Polar and Non-Polar Covalent Bonds

Non-Polar Covalent Bond

When electrons are shared equally between two atoms having the same electronegativity, the bond formed is called a non-polar covalent bond.

Examples: H_2, O_2, N_2

Polar Covalent Bond

When electrons are shared unequally between two atoms with different electronegativities, the bond formed is called a polar covalent bond.

Examples: H_2O, NH_3, HCl

In such molecules, partial positive and partial negative charges develop.

Properties of Covalent Compounds

1. They usually have low melting and boiling points.
2. Most covalent compounds exist as gases, liquids, or soft solids.
3. They are poor conductors of electricity.

4. They are generally insoluble in water but soluble in organic solvents.

Covalent bonding plays a vital role in the formation of molecules essential for life, such as water, oxygen, carbon dioxide, and organic compounds. The sharing of electrons enables atoms to achieve stability without complete transfer of electrons, making covalent bonding one of the most important types of chemical bonding.

1.3 Van-der Waals Bonding

In addition to strong chemical bonds such as ionic and covalent bonds, atoms and molecules also experience **weak intermolecular forces**. These weak attractive forces play a significant role in determining the physical properties of substances, especially gases, liquids, and molecular solids. Such weak forces of attraction are collectively known as

Van der Waals forces, named after the scientist *Johannes Diderik van der Waals*.

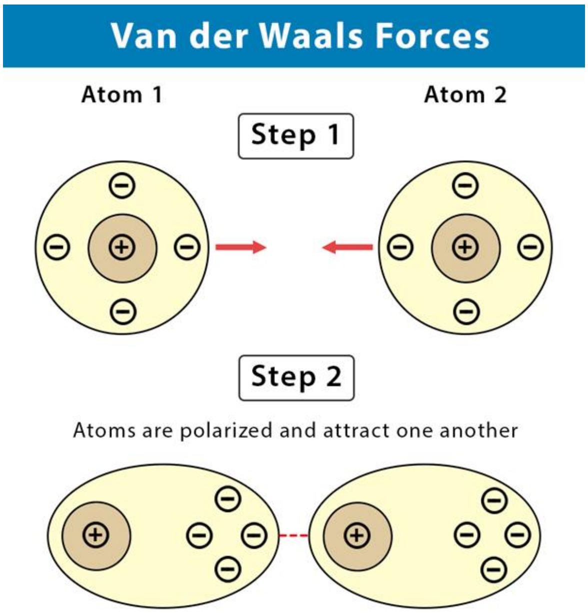


Figure 1. 3

Van der Waals bonding refers to the weak attractive forces that exist between neutral atoms or molecules due to temporary or permanent electric dipoles, without the sharing or transfer of electrons.

Nature of Van der Waals Forces

- They are **much weaker** than ionic or covalent bonds.

- They act over **short distances**.
- They are significant only when atoms or molecules are very close to each other.
- These forces are responsible for the **condensation of gases into liquids** and the formation of **molecular crystals**.

Types of Van der Waals Forces

1. Dipole–Dipole Interaction

This force exists between molecules having **permanent dipoles**. The positive end of one polar molecule attracts the negative end of another polar molecule.

Example:
HCl, SO_2

2. Dipole–Induced Dipole Interaction

A polar molecule induces a temporary dipole in a nearby non-polar molecule, resulting in an attractive force between them.

Example:
O_2 attracted by H_2O

3. London Dispersion Forces

These forces arise due to **temporary fluctuations** in the electron cloud of atoms or molecules, creating instantaneous dipoles. They are present in **all atoms and molecules**, but are the **only forces** acting in non-polar substances.

Example:
Noble gases (He, Ne, Ar), I_2

Van der Waals Bonding in Solids

In certain solids, especially **molecular solids**, particles are held together by Van der Waals forces. These solids are called **Van der Waals solids**.

Examples:

- Solid CO_2 (dry ice)
- Solid iodine
- Solid noble gases at low temperatures

Properties of Van der Waals Solids

1. Low melting and boiling points.
2. Soft in nature.
3. Poor conductors of heat and electricity.
4. Easily compressible.
5. Exist mainly at low temperatures.

Difference Between Covalent Bond and Van der Waals Bonding

Covalent Bond	**Van der Waals Bonding**
Strong bond	Weak attraction
Electron sharing occurs	No electron sharing
Forms molecules	Acts between molecules
High bond energy	Very low bond energy

Importance of Van der Waals Forces

- Responsible for the **liquefaction of gases**.
- Explain the **physical state of molecular substances**.
- Play a crucial role in **biological molecules**, such as protein folding and DNA structure.
- Important in surface phenomena like **adhesion and adsorption**.

Van der Waals bonding represents weak intermolecular attractions that, although small compared to primary chemical bonds, have a major influence on the physical properties of matter. These forces are especially important in understanding the behavior of gases, liquids, and molecular solids.

1.4 Lattice Translational Vectors

A **crystal lattice** is a regular, periodic arrangement of points in three-dimensional space, representing the positions of atoms, ions, or molecules in a crystalline solid. One of the fundamental properties of a crystal lattice is **translational symmetry**. This symmetry is described mathematically using **translational vectors**, which define how the lattice repeats itself in space.

Translational Symmetry

A crystal lattice is said to possess translational symmetry if the lattice remains unchanged when it is displaced by a certain vector. This displacement vector is called a **translation vector**.

If every lattice point, when translated by this vector, coincides with another equivalent lattice point, then the vector represents a valid lattice translation.

Translational Vectors

A **translational vector** is a vector by which the entire crystal lattice can be shifted so that it exactly overlaps with itself.

In a three-dimensional crystal lattice, the position vector **R** of any lattice point can be expressed as:

$$\mathbf{R} = n_1\mathbf{a} + n_2\mathbf{b} + n_3\mathbf{c}$$

where:

- $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are the **primitive translational vectors**
- n_1, n_2, n_3 are integers (positive, negative, or zero)

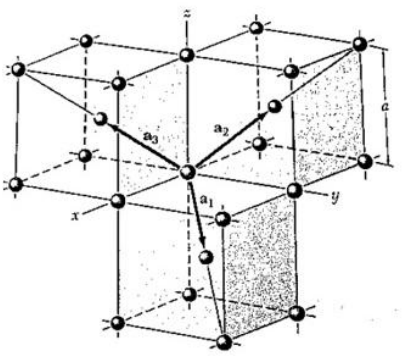
Primitive Translational Vectors

The **primitive translational vectors** are the smallest set of three non-coplanar vectors that can generate the entire crystal lattice through integral linear combinations.

Characteristics:

1. They define the **periodicity** of the crystal.
2. They are not unique; different sets can describe the same lattice.
3. The volume enclosed by these vectors corresponds to the **primitive unit cell**.

Lattice Translation Vector



$$a_1 = \frac{1}{2}a(\hat{x} + \hat{y} - \hat{z})$$

$$a_2 = \frac{1}{2}a(-\hat{x} + \hat{y} + \hat{z})$$

$$a_3 = \frac{1}{2}a(\hat{x} - \hat{y} + \hat{z})$$

Figure 1. 4

Any vector of the form

$$\mathbf{T} = n_1\mathbf{a} + n_2\mathbf{b} + n_3\mathbf{c}$$

is called a **lattice translation vector**.

When a lattice is translated by vector **T**, the arrangement of lattice points remains identical. This property forms the basis of the mathematical description of crystalline solids.

Importance of Translational Vectors

1. They describe the **repeating nature** of crystals.
2. They help define the **unit cell** and crystal structure.

3. They are essential in understanding **X-ray diffraction**, **reciprocal lattice**, and **band theory of solids**.
4. They simplify the study of physical properties by reducing an infinite lattice to a repeating unit.

Relation to Unit Cell

The **unit cell** is the smallest repeating portion of the crystal lattice. Translational vectors determine how the unit cell is repeated throughout space to generate the entire crystal lattice.

- Primitive vectors generate a **primitive unit cell**
- Non-primitive vectors may generate a **conventional unit cell**

Translational vectors form the mathematical foundation of crystal lattice theory. By defining how a lattice repeats itself in space, they provide a precise and systematic way to describe the structure of crystalline solids. Understanding translational vectors is essential for the study of solid-state physics and crystallography.

1.5 Unit Cell

A crystalline solid consists of atoms, ions, or molecules arranged in a regular and repeating pattern in three-dimensional space. This periodic arrangement can be represented by a small repeating unit, which, when translated in all directions, generates the entire crystal lattice. This fundamental repeating unit is known as the **unit cell**.

A **unit cell** is the smallest volume of a crystal lattice which, when repeated continuously in three dimensions by translation, reproduces the complete crystal structure.

Characteristics of a Unit Cell

A unit cell is characterized by:

- **Edge lengths:** a, b, c
- **Interaxial angles:** α, β, γ

These parameters are known as **lattice parameters** and completely describe the geometry of the unit cell.

Types of Unit Cells

1. Primitive Unit Cell

A **primitive unit cell** contains only **one lattice point**. It is the smallest possible unit cell that can represent the lattice.

Features:

- Minimum volume
- Most efficient representation of the lattice
- Not always convenient for visualizing crystal symmetry

2. Conventional Unit Cell

A **conventional unit cell** contains more than one lattice point and is chosen to clearly display the **symmetry** of the crystal.

Features:

- Larger than primitive unit cell
- Easier to visualize and analyze
- Commonly used in crystallography

Unit Cell in Cubic Crystal Systems

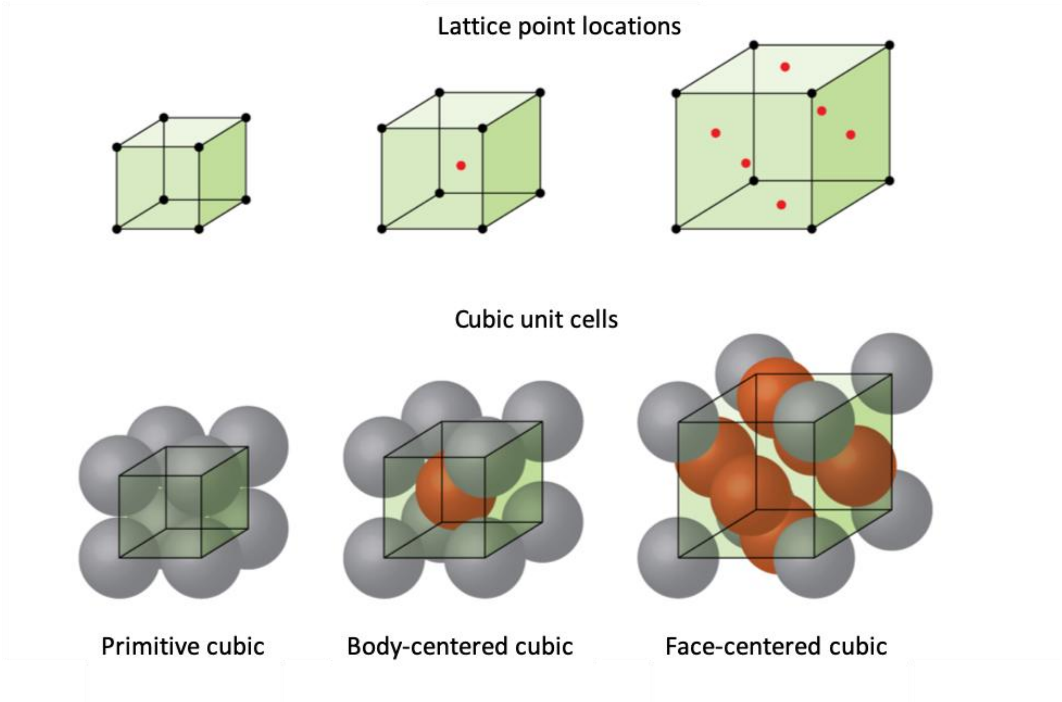


Figure 1. 5

In cubic crystals, unit cells are classified into three types:

1. **Simple Cubic (SC)**
    - Atoms present only at the corners
    - Number of atoms per unit cell = 1
2. **Body-Centred Cubic (BCC)**
    - Atoms at corners and one at the body centre
    - Number of atoms per unit cell = 2
3. **Face-Centred Cubic (FCC)**
    - Atoms at corners and at the centres of all faces
    - Number of atoms per unit cell = 4

Importance of Unit Cell

1. Describes the complete structure of a crystal.
2. Helps in calculating physical properties such as density.
3. Forms the basis for understanding crystal symmetry.
4. Essential for the study of X-ray diffraction and solid-state physics.

Relation Between Unit Cell and Crystal Lattice

- The crystal lattice is an infinite array of lattice points.
- The unit cell is the finite building block of the lattice.
- Repetition of the unit cell by translational symmetry produces the entire crystal lattice.

1.6 Bravais' Lattices

In a crystalline solid, atoms or groups of atoms are arranged in a regular and periodic manner. This regularity can be described using a set of points in space such that each point has an identical environment. Such an arrangement of points is called a **Bravais lattice**. It provides the geometrical framework on which the actual crystal structure is built.

A **Bravais lattice** is an infinite array of points in three-dimensional space in which every lattice point has an identical surrounding environment, and the entire lattice can be generated by translating a single point through a set of discrete translational vectors.

Characteristics of a Bravais Lattice

1. All lattice points are **equivalent**.
2. The lattice possesses **translational symmetry**.
3. Each lattice point represents the position of an atom or a group of atoms (called a basis).

4. The lattice is generated by integral linear combinations of primitive translation vectors.

Crystal Lattice and Basis

A crystal structure can be described as:

$$\text{Crystal Structure} = \text{Bravais Lattice} + \text{Basis}$$

The Bravais lattice gives the periodic arrangement, while the basis specifies the actual atoms associated with each lattice point.

Three-Dimensional Bravais Lattices

In three dimensions, all possible crystal lattices can be classified into **14 distinct Bravais**

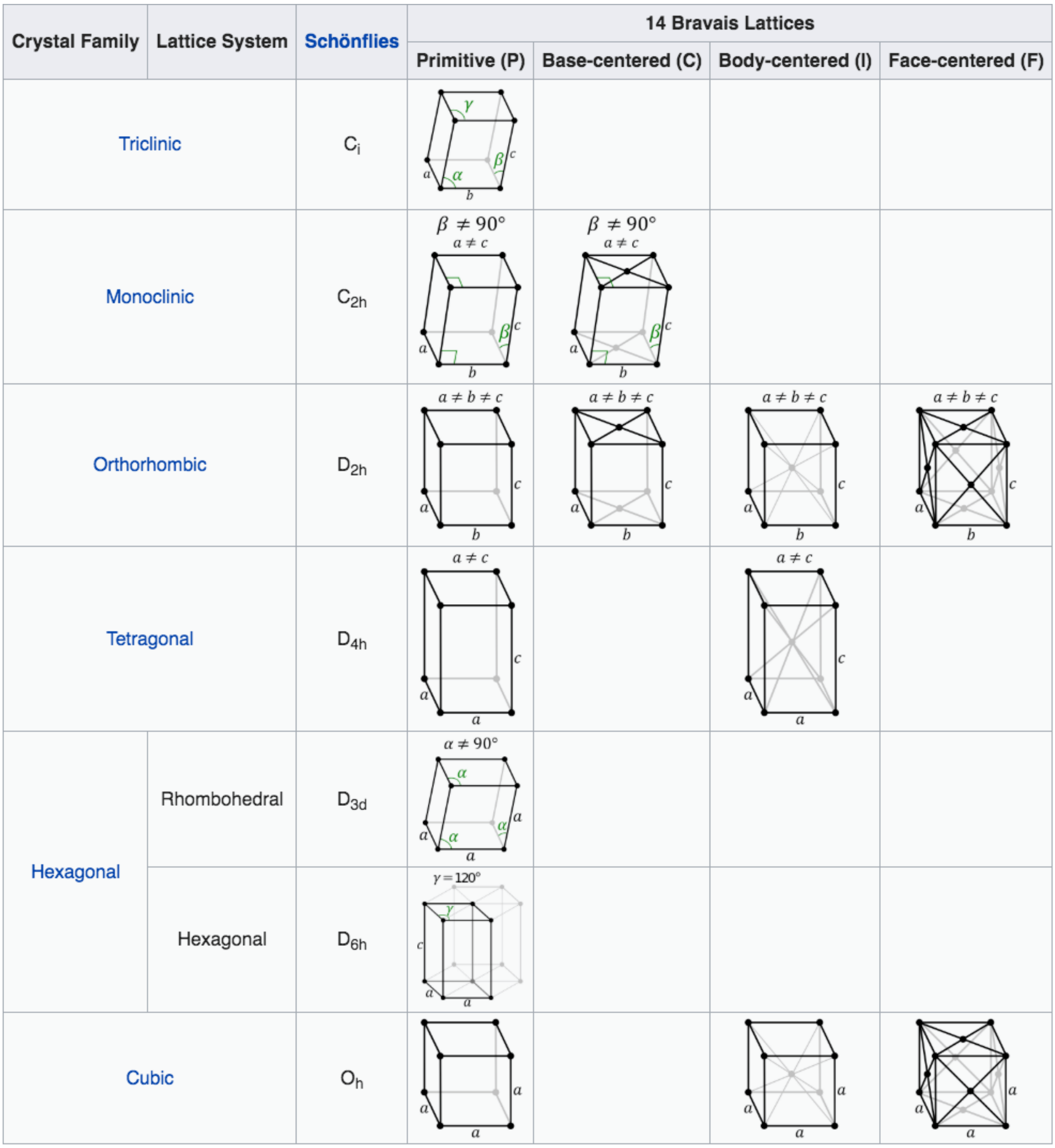


Figure 1.7

lattices. These are grouped into **seven crystal systems** based on their lattice parameters.

Seven Crystal Systems and Bravais Lattices

Crystal System	Bravais Lattices
Cubic	Simple cubic (P), Body-centred cubic (I), Face-centred cubic (F)
Tetragonal	Primitive (P), Body-centred (I)
Orthorhombic	Primitive (P), Body-centred (I), Face-centred (F), Base-centred (C)
Monoclinic	Primitive (P), Base-centred (C)
Triclinic	Primitive (P)
Hexagonal	Primitive (P)
Rhombohedral (Trigonal)	Primitive (P)

Types of Lattice Centering

- **Primitive (P):** Lattice points only at the corners
- **Body-centred (I):** One lattice point at the body centre
- **Face-centred (F):** Lattice points at the centres of all faces
- **Base-centred (C):** Lattice points at the centres of two opposite faces

Importance of Bravais Lattice

1. Provides a complete classification of crystal structures.

2. Forms the foundation of crystallography and solid-state physics.
3. Helps in understanding symmetry, packing, and density of crystals.
4. Essential for the study of X-ray diffraction and electronic properties of solids

The concept of the Bravais lattice is fundamental to the study of crystalline solids. By reducing all possible periodic arrangements to fourteen distinct lattices, Bravais provided a systematic and unified way to describe the geometry of crystal structures.

1.7 Crystal Structure packing of SCC, BCC and FCC

In crystalline solids, atoms are arranged in a regular three-dimensional pattern. The efficiency with which atoms occupy space in a crystal lattice is known as **packing**. The

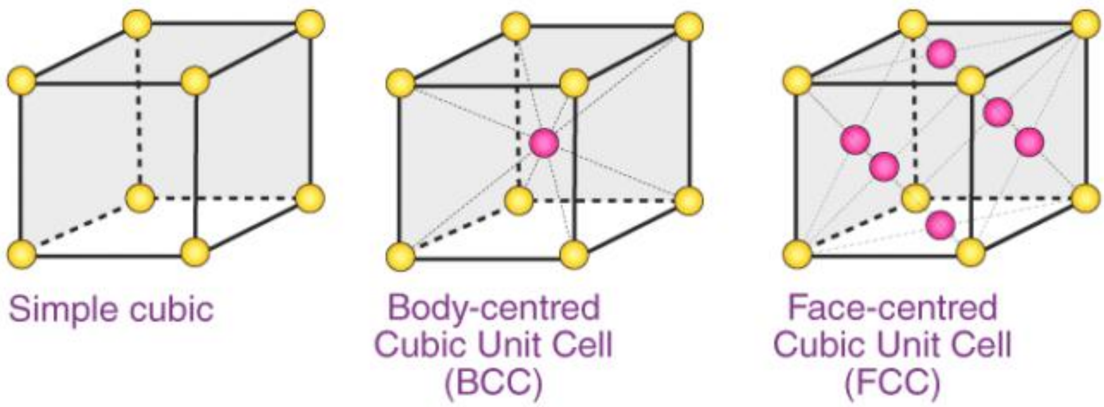


Figure 1. 6

packing of atoms depends on the type of unit cell. Among cubic crystal systems, the most important structures are **Simple Cubic (SC)**, **Body-Centred Cubic (BCC)**, and **Face-Centred Cubic (FCC)**.

1. Simple Cubic Structure (SC)

Arrangement of Atoms

- Atoms are present only at the **eight corners** of the cube.
- Each corner atom is shared by **eight adjacent unit cells**.

Number of Atoms per Unit Cell

$$8 \times \frac{1}{8} = 1 \text{ atom}$$

Coordination Number

- Each atom is in contact with **6 nearest neighbours**.
- Coordination number = **6**

Atomic Radius Relation

Atoms touch each other along the **edge** of the cube.

$$a = 2r$$

where $a$is the edge length and $r$is the atomic radius.

Packing Efficiency

$$\text{Packing efficiency} = \frac{\text{Volume occupied by atoms}}{\text{Volume of unit cell}} \times 100$$

$$= 52\%$$

Example

- Polonium (Po)

2. Body-Centred Cubic Structure (BCC)

Arrangement of Atoms

- Atoms at the **eight corners** of the cube.

- One atom at the **body centre** of the cube.

Number of Atoms per Unit Cell

$$8 \times \frac{1}{8} + 1 = 2 \text{ atoms}$$

Coordination Number

- Each atom is surrounded by **8 nearest neighbours**.
- Coordination number = **8**

Atomic Radius Relation

Atoms touch each other along the **body diagonal**.

$$\sqrt{3}a = 4r$$

Packing Efficiency

$$= 68\%$$

Examples

- Iron (α-Fe)
- Sodium (Na)
- Potassium (K)

3. Face-Centred Cubic Structure (FCC)

Arrangement of Atoms

- Atoms at the **eight corners** of the cube.

- Atoms at the **centres of all six faces**.

Number of Atoms per Unit Cell

$$8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4 \text{ atoms}$$

Coordination Number

- Each atom has **12 nearest neighbours**.
- Coordination number = **12**

Atomic Radius Relation

Atoms touch each other along the **face diagonal**.

$$\sqrt{2}a = 4r$$

Packing Efficiency

$$= 74\%$$

Examples

- Copper (Cu)
- Aluminium (Al)
- Silver (Ag)
- Gold (Au)

Comparison of Packing in SC, BCC and FCC

Property	SC	BCC	FCC
Atoms per unit cell	1	2	4
Coordination number	6	8	12
Packing efficiency	52%	68%	74%
Density	Lowest	Moderate	Highest

1.8 Structures of NaCl and diamond crystal

Crystalline solids are characterized by a definite and regular arrangement of atoms or ions in three-dimensional space. The internal arrangement of particles determines the physical and chemical properties of a crystal. Two important and commonly studied crystal structures are **sodium chloride (NaCl)**, an ionic crystal, and **diamond**, a covalent crystal.

1. Structure of Sodium Chloride (NaCl) Crystal

Nature of Crystal

- NaCl is an **ionic crystal**.
- It consists of **Na^+ and Cl^- ions** held together by strong electrostatic forces of attraction.

Crystal Structure

- NaCl crystallizes in the **face-centred cubic (FCC)** structure.
- The **Cl^- ions** form an FCC lattice.

- The Na^+ **ions** occupy all the **octahedral voids** present in the FCC arrangement of Cl^- ions.

Unit Cell Details

- Number of Cl^- ions per unit cell:

$$8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$$

Number of Na^+ ions per unit cell:

  - 12 edge-centred ions × ¼ = 3
  - 1 body-centred ion × 1 = 1
  - Total Na^+ ions = 4

Coordination Number

- Each Na^+ ion is surrounded by **6 Cl^- ions**.
- Each Cl^- ion is surrounded by **6 Na^+ ions**.
- Coordination number = **6 : 6**

Formula Units per Unit Cell

- Number of NaCl formula units per unit cell = **4**

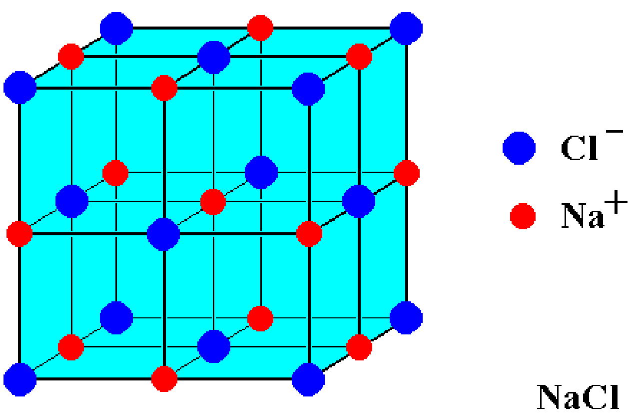


Figure 1. 7

The NaCl crystal structure is also known as the **rock salt structure**. The stability of the crystal is due to strong ionic bonding between oppositely charged ions arranged in a highly symmetrical manner.

2. Structure of Diamond Crystal

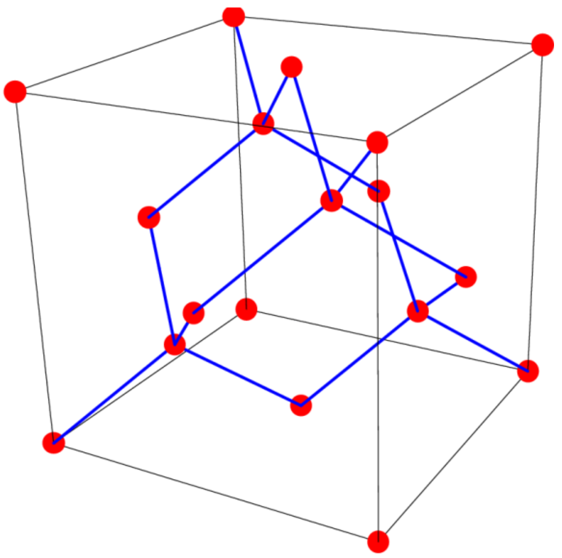
Nature of Crystal

- Diamond is a **covalent crystal**.
- It consists entirely of **carbon atoms** bonded by strong covalent bonds.

Crystal Structure

- Diamond has a **cubic crystal system** with a **face-centred cubic (FCC)** lattice.
- Each carbon atom is covalently bonded to **four other carbon atoms**.
- The bonding arrangement is **tetrahedral**.

Atomic Arrangement



- The structure can be considered as an FCC lattice with a basis of **two carbon atoms**.
- The second carbon atom is located at a position $\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)$relative to the FCC lattice points.

Coordination Number

- Each carbon atom is surrounded by **4 nearest neighbours**.
- Coordination number = **4**

Atoms per Unit Cell

- Total number of carbon atoms per unit cell = **8**

Bonding and Properties

- Strong covalent bonds extend throughout the crystal.
- No free electrons are present.
- This continuous network of covalent bonds gives diamond its:
    - High hardness
    - High melting point
    - Electrical insulating nature

Diamond is a **giant covalent crystal** in which each atom is strongly bonded to its neighbours, resulting in exceptional mechanical strength and rigidity.

Comparison Between NaCl and Diamond Crystal

Property	NaCl Crystal	Diamond Crystal
Type of bonding	Ionic	Covalent
Lattice type	FCC	FCC
Coordination number	6	4
Particles	Ions (Na^+, Cl^-)	Carbon atoms
Electrical conductivity	Conducts in molten state	Insulator
Hardness	Moderate	Very high

The NaCl crystal represents a typical **ionic crystal structure**, while diamond represents a **covalent network crystal**. Both structures highlight how different types of bonding lead to distinctly different physical properties in crystalline solids.

1.9 Procedure for finding Miller Indices

In crystallography, **Miller indices** are a set of three integers $(h\,k\,l)$used to specify the orientation of planes in a crystal lattice. They provide a simple and systematic method to describe crystallographic planes and are widely used in the study of crystal structures, X-ray diffraction, and solid-state physics.

Step-by-Step Procedure

Step 1: Identify the Intercepts of the Plane

Determine the points at which the given crystal plane intersects the crystallographic axes x, y, and z.

Let the intercepts be expressed in terms of the unit cell dimensions a, b, and c.

Step 2: Express the Intercepts as Fractions

Write the intercepts in the form:

$$\left(\frac{p}{a}, \frac{q}{b}, \frac{r}{c}\right)$$

If a plane is parallel to any axis, its intercept along that axis is **infinite (∞)**.

Step 3: Take the Reciprocals of the Intercepts

Find the reciprocals of the intercepts obtained in Step 2.

$$\left(\frac{a}{p}, \frac{b}{q}, \frac{c}{r}\right)$$

The reciprocal of ∞is **zero**.

Step 4: Clear the Fractions

Multiply the reciprocals by the **least common multiple (LCM)** to obtain the smallest set of **whole numbers**.

Step 5: Write the Miller Indices

The resulting integers are written as $(h\ k\ l)$and represent the **Miller indices** of the given plane.

Special Cases

1. If a plane cuts only one axis, the Miller index corresponding to the other axes will be **zero**.
2. Negative intercepts are indicated by placing a **bar** over the corresponding Miller index.
    - Example: $(\bar{1}\ 0\ 1)$
3. Miller indices are always expressed as **integers**, not fractions.

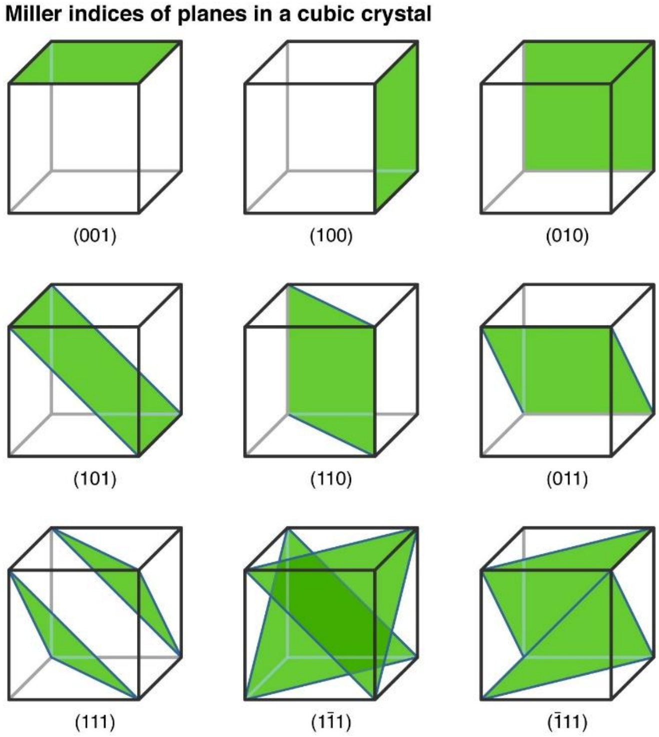


Figure 1. 8

Illustrative Example

If a plane cuts the axes at:

$$2a,\ 3b,\ \infty c$$

Intercepts: $(2,\ 3,\ \infty)$

- Reciprocals: $\left(\frac{1}{2},\ \frac{1}{3},\ 0\right)$
- Clearing fractions (LCM = 6): $(3,\ 2,\ 0)$

$$\therefore \text{Miller indices} = (3\ 2\ 0)$$

Importance of Miller Indices

1. Used to represent crystal planes compactly.
2. Essential for interpreting X-ray diffraction patterns.
3. Help in understanding crystal symmetry and structure.
4. Important in solid-state physics and materials science.

Miller indices provide a standardized and efficient method to describe the orientation of planes in a crystal lattice. By following a simple procedure involving intercepts, reciprocals, and integer conversion, any crystallographic plane can be uniquely identified.

Unit 2: Elementary Lattice Dynamics

1. Introduction
2. Lattice vibrations and phonons
3. Linear monoatomic and diatomic chains
4. Acoustical and optical phonons
5. Dulonga Petit's Law
6. Properties of metals
7. Classical free Electron theory of metals (Drude-Lorentz)
8. Ohm's Law
9. Electrical and thermal conductivities
10. Weidemann-Franz' law

2.1 Introduction

In a crystalline solid, atoms are not rigidly fixed at their lattice points. Instead, they execute **small oscillations about their equilibrium positions** due to thermal energy. The study of these atomic motions and their consequences is known as **lattice dynamics**. Elementary lattice dynamics deals with the **basic concepts of vibrations of atoms in a crystal lattice** and provides a foundation for understanding several thermal and electrical properties of solids.

Need for Studying Lattice Dynamics

The physical properties of solids such as:

- Specific heat
- Thermal conductivity

- Elastic behavior
- Interaction of solids with radiation

cannot be explained by assuming atoms to be stationary. These properties arise due to the **vibrational motion of atoms** in the lattice, making lattice dynamics an essential topic in solid-state physics.

Basic Assumptions

In elementary lattice dynamics, the following assumptions are generally made:

1. Atoms are arranged in a **perfect periodic lattice**.
2. Each atom oscillates about its mean equilibrium position.
3. Interatomic forces act as **restoring forces** when atoms are displaced.
4. The displacements are small, allowing the use of **harmonic approximation**.

Nature of Atomic Vibrations

When an atom in a crystal is displaced from its equilibrium position, restoring forces arise due to interactions with neighboring atoms. As a result, the atom vibrates about its mean position. These vibrations are not isolated; they are **collective in nature**, involving many atoms moving together in a coordinated manner.

Lattice Vibrations and Phonons

The collective vibrational modes of the crystal lattice can be described in terms of quantized energy packets called **phonons**. Phonons are the quanta of lattice vibrations and play a role similar to photons in electromagnetic radiation.

Importance of Elementary Lattice Dynamics

1. Explains the **origin of heat capacity** of solids.
2. Helps in understanding **thermal expansion** and **thermal conductivity**.
3. Forms the basis for studying **phonons and dispersion relations**.

4. Essential for advanced topics such as **electron–phonon interaction** and **superconductivity**.

2.2 Lattice vibrations and phonons

In a crystalline solid, atoms are arranged in a periodic lattice. These atoms are not stationary; they continuously execute small oscillations about their equilibrium positions due to thermal energy. The collective and coordinated motion of atoms in a crystal lattice is known as **lattice vibrations**. The quantum mechanical description of these vibrations leads to the concept of **phonons**, which play a crucial role in determining the thermal, electrical, and optical properties of solids.

Lattice Vibrations

Origin of Lattice Vibrations

Atoms in a crystal are bound to each other by interatomic forces. When an atom is displaced from its equilibrium position, restoring forces arise due to interactions with neighboring atoms. As a result, the atom oscillates about its mean position. Since atoms are coupled, the vibration of one atom affects others, leading to **collective vibrational motion** throughout the lattice.

Harmonic Approximation

For small displacements, the restoring force acting on an atom is proportional to its displacement. This approximation is called the **harmonic approximation**. Under this assumption:

- The vibrations are simple and periodic.
- The potential energy is quadratic in displacement.
- The equations of motion become mathematically tractable.

Normal Modes of Vibration

A crystal containing a large number of atoms has many possible vibrational patterns. Each independent pattern of vibration, in which all atoms oscillate with the same frequency, is called a **normal mode** of vibration. The total lattice vibration can be expressed as a superposition of these normal modes.

Phonons

Concept of Phonons

In classical mechanics, lattice vibrations are treated as waves. However, according to quantum mechanics, vibrational energy is quantized. Each quantum of lattice vibrational energy is called a **phonon**.

A phonon is defined as the **quantized mode of vibration of atoms in a crystal lattice**.

Energy of a Phonon

The energy associated with a phonon of frequency $\nu$is given by:

$$E = h\nu$$

where $h$is Planck's constant.

Phonons are **bosons** and do not carry electric charge. They represent collective excitations

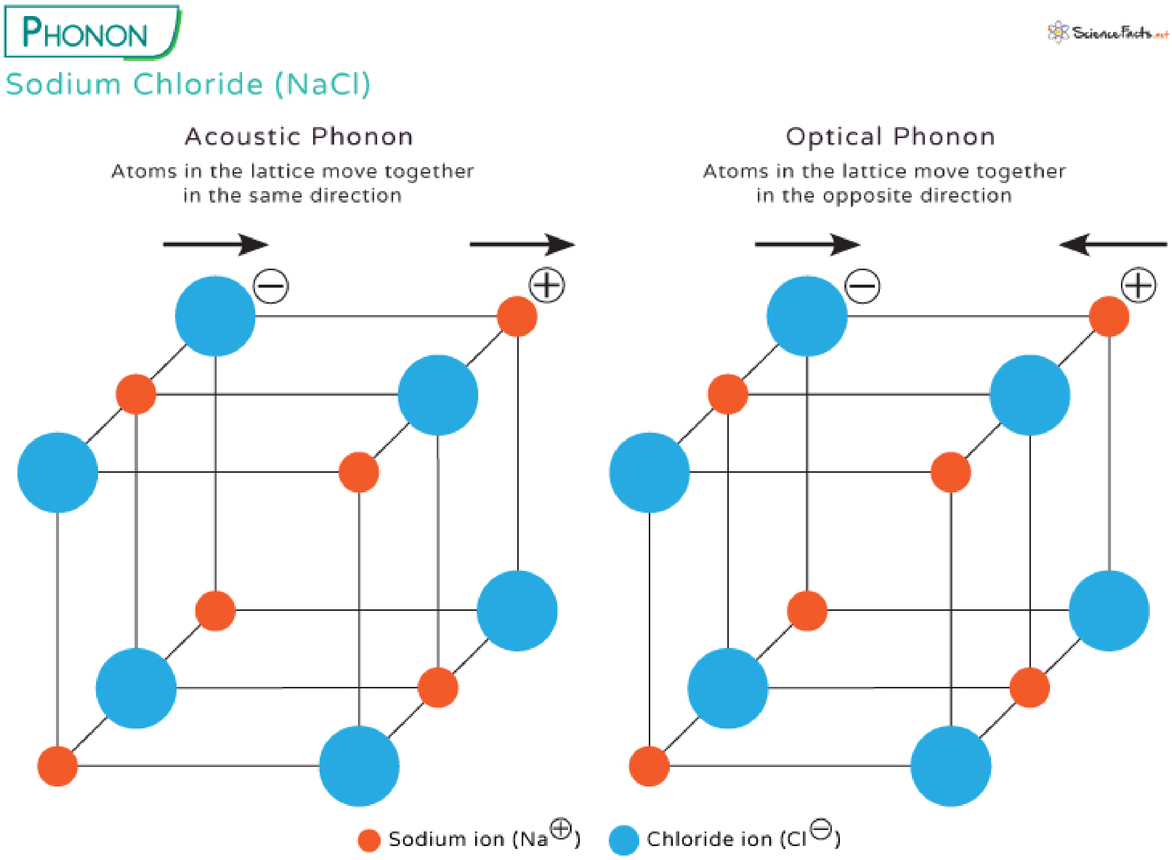


Figure 2. 1

rather than individual particles.

Types of Phonons

1. Acoustic Phonons

- Arise from vibrations in which neighboring atoms oscillate **in phase**.
- At long wavelengths, these vibrations resemble sound waves.
- Responsible for the **propagation of sound** and **thermal conductivity** in solids.
- Frequency approaches zero as wavelength becomes very large.

2. Optical Phonons

- Occur in crystals with **more than one atom per primitive unit cell**.

- Neighboring atoms oscillate **out of phase** with each other.
- Have higher frequencies compared to acoustic phonons.
- Can interact strongly with electromagnetic radiation, especially in the infrared region.

Phonon Dispersion Relation

The relationship between phonon frequency $\omega$and wave vector $k$is known as the **phonon dispersion relation**.

Key points:

- Acoustic branches start from zero frequency.
- Optical branches start at finite frequencies.
- Dispersion curves provide information about vibrational modes and lattice stability.

Role of Phonons in Physical Properties

1. Specific Heat

Phonons are the primary contributors to the heat capacity of insulating solids. At low temperatures, phonon behavior explains the deviation from classical predictions.

2. Thermal Conductivity

Heat is transported in insulating solids mainly by phonons. Phonon scattering determines the thermal conductivity.

3. Electrical Properties

In conductors and semiconductors, phonons interact with electrons, affecting electrical resistivity and mobility.

4. Optical Properties

Optical phonons are involved in infrared absorption and Raman scattering experiments.

2.3 Linear monoatomic and diatomic chains

To understand lattice vibrations in crystalline solids, simple one-dimensional models are often used. Among these, the **linear monoatomic chain** and **linear diatomic chain** are fundamental models. Though idealized, they provide clear insight into how atoms vibrate collectively in a lattice and how vibrational waves propagate through solids.

1. Linear Monoatomic Chain

Description of the Model

A linear monoatomic chain consists of:

- Identical atoms of mass m
- Arranged at equal distances a
- Confined to move only along one dimension
- Interacting with nearest neighbors through restoring forces

The interatomic force is assumed to obey **Hooke's law**, characterized by a force constant α.

Equation of Motion

When atoms are displaced slightly from equilibrium, restoring forces act due to neighboring atoms. The equation of motion for the n-th atom leads to a wave-like solution, indicating that lattice vibrations propagate as waves through the chain.

Dispersion Relation

The relation between angular frequency $\omega$and wave vector $k$is given by:

$$\omega = 2\sqrt{\frac{\alpha}{m}} \mid \sin\frac{ka}{2} \mid$$

Important Features

- Only **one branch** of vibrations exists.

- Frequency becomes zero when $k \to 0$.
- These vibrations correspond to **acoustic phonons**.

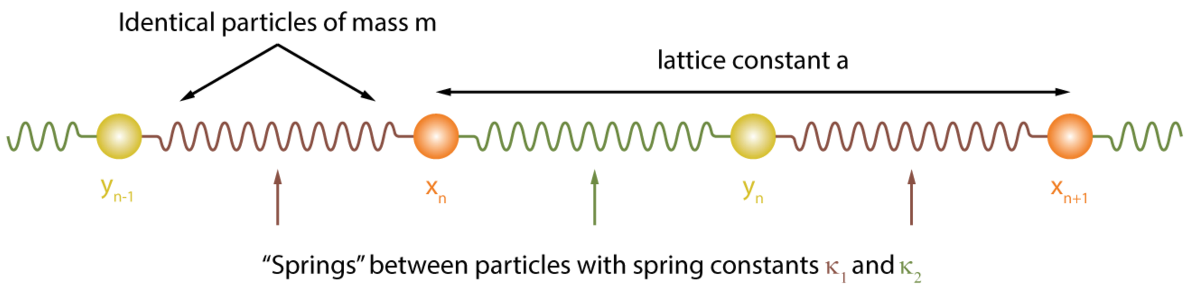


Figure 2. 2

- Long-wavelength vibrations resemble sound waves.

Physical Significance

The monoatomic chain explains:

- Propagation of sound in solids
- Acoustic phonon behavior
- Basic concept of dispersion in lattices

2. Linear Diatomic Chain

Description of the Model

A linear diatomic chain consists of:

- Two different atoms with masses $m_1$and m_2
- Arranged alternately along a line
- Equal spacing between nearest neighbors
- Nearest-neighbor interactions with force constant α

This model better represents **real crystals** with more than one atom per primitive unit cell.

Equation of Motion

Due to two different masses, the equations of motion lead to **two distinct vibrational modes** for each wave vector.

Dispersion Relation

The solution gives **two branches**:

- **Lower frequency branch**
- **Higher frequency branch**

Types of Vibrational Modes

1. Acoustic Mode

- Atoms oscillate **in phase**
- Frequency approaches zero as $k \to 0$
- Responsible for sound propagation

2. Optical Mode

- Atoms oscillate **out of phase**
- Frequency is finite even at $k = 0$
- Can interact with electromagnetic radiation

Frequency Gap

A **frequency gap** exists between acoustic and optical branches, especially when the masses m_1 and m_2 differ significantly. This gap has important consequences in thermal and optical properties.

Comparison Between Monoatomic and Diatomic Chains

Feature	Monoatomic Chain	Diatomic Chain
Atoms per unit cell	1	2
Number of branches	1	2
Types of modes	Acoustic only	Acoustic + Optical
Frequency at $k = 0$	Zero	Zero and finite
Real crystal representation	Idealized	More realistic

Importance of Linear Chain Models

1. Provide a simple understanding of lattice vibrations.
2. Explain the origin of **phonon dispersion relations**.
3. Help distinguish between **acoustic and optical phonons**.
4. Form the basis for studying thermal and optical properties of solids.

The linear monoatomic chain introduces the basic concept of collective atomic vibrations and acoustic phonons, while the linear diatomic chain extends this understanding by introducing optical modes and frequency gaps. Together, these models form the foundation for lattice dynamics and phonon theory in solid-state physics.

2.4 Acoustical and optical phonons

In a crystalline solid, atoms execute collective vibrations about their equilibrium positions. When these lattice vibrations are described using quantum mechanics, they are represented in terms of **phonons**. Depending on the nature of atomic motion and the frequency of vibration, phonons are classified mainly into **acoustical phonons** and **optical phonons**. This classification is particularly important in crystals containing more than one atom per primitive unit cell.

Acoustical Phonons

Acoustical phonons are vibrational modes in which neighboring atoms in the lattice oscillate **in phase** with each other.

Characteristics

1. The frequency of acoustical phonons approaches **zero** as the wavelength becomes very large.
2. These phonons correspond to **sound waves** propagating through the crystal.
3. They are present in **all crystalline solids**, including monoatomic lattices.
4. Acoustical phonons have **low energy** and **long wavelength**.

Physical Role

- Responsible for the **propagation of sound** in solids.
- Major contributors to **thermal conductivity**.
- Play a significant role in elastic properties of crystals.

Optical Phonons

Optical phonons are vibrational modes in which neighboring atoms in the lattice oscillate **out of phase** with each other.

Characteristics

1. Optical phonons have **finite frequency** even at zero wave vector.
2. They occur only in crystals having **two or more atoms per primitive unit cell**.
3. These phonons have **higher energy** than acoustical phonons.
4. They can interact strongly with **electromagnetic radiation**, especially in the infrared region.

Physical Role

- Responsible for **infrared absorption** and **Raman scattering**.
- Influence dielectric and optical properties of solids.
- Affect electron–phonon interactions in semiconductors.

Dispersion Relation

The relationship between phonon frequency and wave vector reveals two distinct branches in diatomic lattices:

- **Acoustical branch** starting from zero frequency.
- **Optical branch** starting from a finite frequency.

This dispersion behavior explains the existence and properties of both acoustical and optical phonons.

Comparison Between Acoustical and Optical Phonons

Feature	Acoustical Phonons	Optical Phonons
Atomic motion	In phase	Out of phase
Frequency at $k = 0$	Zero	Finite
Occurrence	All crystals	Multi-atomic crystals
Energy	Low	High

Interaction with light	Weak	Strong
Role in heat conduction	Major	Minor

Acoustical and optical phonons represent two fundamental types of lattice vibrations in crystalline solids. Acoustical phonons govern sound propagation and thermal transport, while optical phonons are crucial for understanding the optical and dielectric behavior of materials. Together, they provide a complete picture of lattice dynamics in solids.

2.5 Dulonga Petit's Law

The study of heat capacity of solids is an important part of solid-state physics. Early experimental studies showed that many solid elements exhibit nearly the same molar heat capacity at ordinary temperatures. In 1819, **Pierre Louis Dulong** and **Alexis Thérèse Petit** proposed a simple empirical law to explain this observation, which is known as **Dulong and Petit's law**.

Statement of Dulong and Petit's Law

According to **Dulong and Petit's law**:

The molar specific heat capacity of a solid element is approximately constant and equal to 3R.

Mathematically,

$$C_v = 3R$$

where

- C_v= molar specific heat at constant volume
- R= universal gas constant

Since $R \approx 8.314 \text{ J mol}^{-1}\text{K}^{-1}$,

$$C_v \approx 25 \text{ J mol}^{-1}\text{K}^{-1}$$

Explanation of the Law

In a solid, atoms are bound to fixed equilibrium positions and can vibrate about these positions. According to classical physics:

- Each atom has **three degrees of freedom** corresponding to motion along the x, y, and z directions.
- Each degree of freedom contributes an energy of $\frac{1}{2}kT$ as kinetic energy and $\frac{1}{2}kT$ as potential energy.
- Thus, total energy per atom is $3kT$.

For one mole of atoms:

$$U = 3RT$$

Differentiating with respect to temperature,

$$C_v = \frac{dU}{dT} = 3R$$

Significance of Dulong and Petit's Law

1. It was one of the earliest attempts to explain the thermal behavior of solids.
2. It provided strong support for the **atomic theory of matter**.
3. It gives a good approximation for the molar heat capacity of **many solids at high temperatures**.
4. It helped in estimating **atomic weights** of elements.

Limitations of the Law

1. The law fails at **low temperatures**.
2. It does not apply to **light elements** such as carbon, boron, and silicon.
3. It cannot explain why specific heat varies with temperature.

Figure 2. 3

4. It is based on **classical physics**, which neglects quantum effects.

Experimental Observations

- For many metals like copper, silver, and aluminium, the molar heat capacity is close to $3R$at room temperature.
- For substances like diamond, the molar heat capacity is much smaller at ordinary temperatures and increases with temperature.

Dulong and Petit's law states that the molar heat capacity of solid elements is nearly constant and equal to $3R$. Although the law successfully explains the behavior of many solids at high temperatures, it fails at low temperatures and for light elements. These limitations led to the development of improved theories such as **Einstein's theory** and **Debye's theory** of specific heat.

2.6 Properties of metals

In metallic solids, atoms are arranged in a regular crystal lattice, where positively charged metal ions occupy lattice points and valence electrons are free to move throughout the crystal. The study of **lattice dynamics in metals** deals with the vibrational motion of metal ions and their interaction with free electrons. These lattice vibrations significantly influence the **thermal, electrical, and mechanical properties** of metals.

Lattice Vibrations in Metals

In metals, the ions at lattice points vibrate about their equilibrium positions due to thermal energy. These vibrations are collective in nature and are described in terms of **phonons**. Unlike insulators, lattice vibrations in metals strongly interact with free electrons.

Important Properties of Metals Related to Lattice Dynamics

1. Electrical Conductivity

Metals exhibit high electrical conductivity due to the presence of free electrons.

- Lattice vibrations scatter conduction electrons.
- As temperature increases, lattice vibrations increase.
- Increased electron–phonon scattering leads to a **decrease in electrical conductivity** with rise in temperature.

2. Thermal Conductivity

Heat conduction in metals occurs mainly due to:

- Free electrons (dominant contribution)
- Phonons (lattice vibrations)

At higher temperatures, enhanced lattice vibrations cause more scattering, affecting thermal transport.

3. Temperature Dependence of Resistivity

The electrical resistivity of metals increases with temperature because:

- Lattice vibrations become more intense.
- The probability of electron–phonon collisions increases.
- This impedes the motion of electrons through the lattice.

4. Specific Heat of Metals

The total specific heat of metals has two contributions:

1. **Lattice contribution** due to phonons.
2. **Electronic contribution** due to free electrons.

At ordinary temperatures, lattice vibrations dominate the specific heat, while at very low temperatures, electronic contribution becomes significant.

5. Elastic Properties

The elastic behavior of metals depends on:

- Interatomic forces between ions.

- Response of lattice vibrations to applied stress.

Lattice dynamics helps explain elastic constants and sound propagation in metals.

6. Sound Propagation

Sound waves in metals propagate as **acoustic phonons**.

- Their velocity depends on lattice stiffness and ionic mass.
- Lattice dynamics explains the relation between elastic constants and sound velocity.

7. Electron–Phonon Interaction

One of the most important aspects of lattice dynamics in metals is **electron–phonon interaction**.

- It affects electrical resistivity.
- Plays a crucial role in **superconductivity**.
- Influences optical and thermal properties.

8. Thermal Expansion

As temperature increases:

- Amplitude of lattice vibrations increases.
- Average separation between ions increases.
- This leads to **thermal expansion** of metals.

Importance of Lattice Dynamics in Metals

1. Explains temperature dependence of electrical and thermal properties.
2. Helps understand scattering mechanisms in metals.
3. Essential for studying superconductivity.
4. Provides insight into mechanical strength and elasticity.

The lattice dynamics of metals involves the study of vibrations of metal ions and their interaction with free electrons. These lattice vibrations play a central role in determining the electrical conductivity, thermal conductivity, specific heat, elasticity, and resistivity of metals. Hence, understanding lattice dynamics is essential for explaining the macroscopic physical properties of metallic solids.

2.7 Classical free Electron theory of metals (Drude-Lorentz)

The electrical and thermal properties of metals attracted scientific attention long before the development of quantum mechanics. To explain these properties, **Paul Drude (1900)** and later **H. A. Lorentz** proposed the **classical free electron theory of metals**. This theory treats the conduction electrons in a metal as a gas of free particles moving randomly inside the metal, obeying the laws of classical mechanics.

Basic Assumptions of Drude–Lorentz Theory

The theory is based on the following assumptions:

1. A metal consists of a large number of **positive ions fixed at lattice points**.
2. Valence electrons are **free electrons** and can move throughout the entire volume of the metal.
3. Free electrons behave like an **ideal gas** and obey classical Newtonian mechanics.
4. Electron–electron interactions are neglected.
5. Electrons collide occasionally with ions or imperfections in the lattice.
6. Collisions are instantaneous and random, and after each collision electrons lose memory of their previous motion.
7. Between collisions, electrons move freely under the influence of external fields.

Motion of Free Electrons

In the absence of an external electric field, electrons move randomly in all directions with thermal velocities. Hence, the net current is zero.

When an electric field **E**is applied:

- Each electron experiences a force $-e\mathbf{E}$.
- Electrons acquire a small **drift velocity** opposite to the direction of the electric field.
- This drift of electrons gives rise to **electric current**.

Electrical Conductivity

According to the Drude model, electrical conductivity $\sigma$is given by:

$$\sigma = \frac{ne^2\tau}{m}$$

where

- n= number of free electrons per unit volume
- e= electronic charge
- τ= mean relaxation time
- m= mass of electron

This expression successfully explains:

- Ohm’s law
- Dependence of conductivity on electron density

Thermal Conductivity

The theory assumes that free electrons also transport heat energy. Thus:

- Metals show high thermal conductivity.
- Thermal and electrical conductivities are related.

This leads to the **Wiedemann–Franz law**, which states that the ratio of thermal conductivity to electrical conductivity is proportional to temperature.

Successes of Drude–Lorentz Theory

1. Explains **Ohm's law**.
2. Accounts for **high electrical conductivity** of metals.
3. Explains **thermal conductivity** of metals.
4. Provides a qualitative explanation of the **Wiedemann–Franz law**.
5. Explains the effect of temperature on resistivity of metals.

Limitations of Drude–Lorentz Theory

1. Fails to explain the **temperature dependence of specific heat** of metals.
2. Cannot explain why some solids are **insulators or semiconductors**.
3. Gives incorrect values for **mean free path** of electrons.
4. Cannot explain **Hall effect** correctly for all metals.
5. Ignores quantum nature of electrons.

Lorentz Modification

Lorentz refined Drude's theory by including:

- Statistical distribution of electron velocities
- Better treatment of collisions

However, the theory still remained **classical** and could not fully explain experimental observations.

The classical free electron theory of metals proposed by Drude and Lorentz was a pioneering attempt to explain the electrical and thermal properties of metals. Although it successfully explained several macroscopic properties, its failure to account for quantum effects led to the development of **quantum free electron theory** and **band theory of solids**.

Despite its limitations, the Drude–Lorentz theory remains an important milestone in solid-state physics.

2.8 Ohm's Law

Ohm's law describes the relationship between current, potential difference, and resistance in a conductor. In the context of **crystal lattice dynamics**, Ohm's law is explained by considering the motion of **free electrons through a vibrating crystal lattice** of positive ions. The electrical behavior of metals arises from the interaction between moving electrons and the lattice vibrations of ions.

Statement of Ohm's Law

Ohm's law states that, at constant temperature, the current flowing through a conductor is directly proportional to the potential difference applied across it.

$$V \propto I \Rightarrow V = IR$$

where

- V= potential difference
- I= electric current
- R= electrical resistance

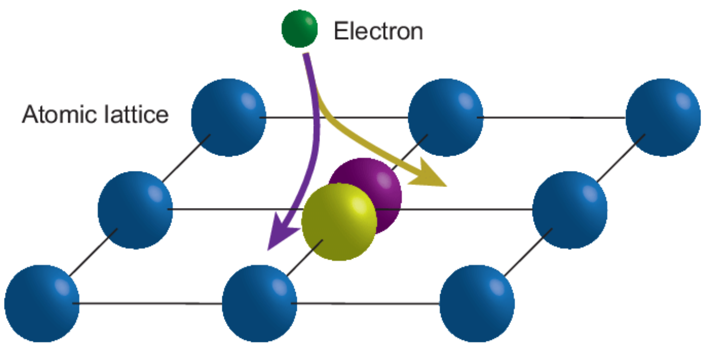


Figure 2. 4

Microscopic View of Ohm's Law

In a metal crystal:

- Positive ions are arranged periodically at lattice points.
- Valence electrons move freely through the lattice.
- In the absence of an electric field, electrons move randomly, producing no net current.

When an external electric field **E**is applied:

- Electrons experience a force opposite to the field.
- They acquire a small **drift velocity** superimposed on their random motion.
- This drift motion results in an electric current.

Role of Lattice Vibrations

At finite temperatures:

- Ions in the crystal lattice vibrate about their equilibrium positions.
- These vibrations are described as **phonons**.
- Moving electrons collide with vibrating ions (electron–phonon interaction).

These collisions:

- Limit the drift velocity of electrons.
- Cause resistance to the flow of current.
- Ensure that current remains proportional to the applied electric field.

Electrical Conductivity and Lattice Dynamics

The electrical conductivity $\sigma$of a metal is given by:

$$\sigma = \frac{ne^2\tau}{m}$$

where

- n= number of free electrons per unit volume
- e= charge of electron
- τ= mean relaxation time between collisions
- m= mass of electron

Lattice vibrations reduce $\tau$by increasing collision frequency, thereby reducing conductivity.

Temperature Dependence

- As temperature increases, lattice vibrations increase.
- Electron–phonon scattering becomes more frequent.
- Electrical resistance increases.
- Ohm's law remains valid, but the value of resistance changes with temperature.

Validity of Ohm's Law

Ohm's law holds when:

1. Electric field is not extremely strong.
2. Temperature of the conductor is constant.
3. Electron motion remains in the linear response regime.

Under these conditions, the crystal lattice responds linearly to the applied field.

Importance in Solid-State Physics

1. Provides a microscopic explanation of electrical conduction.
2. Connects macroscopic electrical laws with lattice dynamics.
3. Helps understand resistivity and its temperature dependence.
4. Forms the basis for electron transport theory in solids.

In crystal lattice dynamics, Ohm's law arises from the drift motion of free electrons through a vibrating lattice of ions. The proportionality between current and applied electric field is maintained due to electron–phonon scattering, which limits electron acceleration and establishes steady-state conduction. Thus, lattice vibrations play a central role in explaining Ohm's law at the microscopic level.

2.9 Electrical and thermal conductivities

Electrical and thermal conductivities are two important transport properties of solids. In crystalline materials, both these properties are governed by the motion of **electrons** and the vibrations of the **crystal lattice (phonons)**. While electrical conductivity is mainly due to free electrons, thermal conductivity arises due to both electrons and lattice vibrations.

1. Electrical Conductivity

Electrical conductivity is the ability of a material to conduct electric current under the influence of an applied electric field.

$$\sigma = \frac{J}{E}$$

where

- σ= electrical conductivity
- J= current density
- E= electric field

In metals:

- Positive ions are fixed at lattice points.
- Valence electrons move freely through the crystal.
- When an electric field is applied, electrons acquire a drift velocity.

The conductivity is given by:

$$\sigma = \frac{ne^2\tau}{m}$$

where

- n= number of free electrons per unit volume
- e= charge of electron
- τ= relaxation time
- m= mass of electron

Role of Lattice Vibrations

- Lattice vibrations (phonons) scatter electrons.
- Increased vibrations reduce relaxation time.
- This scattering gives rise to electrical resistance.

Temperature Dependence

- In metals, electrical conductivity decreases with increase in temperature.
- This is due to increased electron–phonon scattering.
- In semiconductors, conductivity increases with temperature due to increased carrier concentration.

2. Thermal Conductivity

Definition

Thermal conductivity is the ability of a material to conduct heat.

$$K = \frac{Q}{A\,(dT/dx)}$$

where

- K= thermal conductivity
- Q= heat flow per unit time
- A= cross-sectional area

- dT/dx= temperature gradient

Mechanisms of Heat Conduction

Thermal conductivity in solids occurs due to:

1. **Electrons**
2. **Phonons (lattice vibrations)**

$$K = K_e + K_p$$

where

- K_e= electronic contribution
- K_p= lattice (phonon) contribution

Thermal Conductivity in Metals

- Free electrons carry both charge and heat.
- Electronic thermal conductivity dominates.
- Strongly influenced by electron–phonon interactions.

Thermal Conductivity in Insulators

- Very few free electrons.
- Heat is mainly carried by phonons.
- Lattice imperfections and phonon–phonon collisions reduce heat flow.

Temperature Dependence

- At low temperatures, thermal conductivity increases due to reduced scattering.

- At high temperatures, phonon–phonon collisions dominate, reducing thermal conductivity.

3. Wiedemann–Franz Law

In metals, electrical and thermal conductivities are related by the **Wiedemann–Franz law**:

$$\frac{K}{\sigma T} = L$$

where

- T= absolute temperature
- L= Lorenz number

This law shows that good electrical conductors are also good thermal conductors.

Importance in Solid-State Physics

1. Explains heat and charge transport in crystals.
2. Essential for understanding metals, semiconductors, and insulators.
3. Forms the basis for thermal management in electronic devices.
4. Helps in material selection for electrical and thermal applications.

Electrical conductivity in solids is mainly due to the drift of free electrons through the crystal lattice, while thermal conductivity arises from both electrons and lattice vibrations. The interaction between electrons and phonons plays a crucial role in determining both properties, linking crystal lattice dynamics with macroscopic transport phenomena.

2.10 Weidemann-Franz' law

The Wiedemann–Franz law establishes a fundamental relationship between the **electrical conductivity** and **thermal conductivity** of metals. It arises from the fact that, in metals, **free electrons** are responsible for carrying both electric charge and heat. This law is an important result of the **classical free electron theory** and plays a key role in crystal lattice dynamics and transport phenomena.

Statement of the Law

The Wiedemann–Franz law states that:

For a given metal, the ratio of thermal conductivity to the product of electrical conductivity and absolute temperature is constant.

Mathematically,

$$\frac{K}{\sigma T} = L$$

where

- K= thermal conductivity
- σ= electrical conductivity
- T= absolute temperature
- L= Lorenz number

Lorenz Number

The constant $L$is called the **Lorenz number**.

For most metals, its theoretical value is:

$$L = 2.44 \times 10^{-8}\ \mathrm{W\,\Omega\,K^{-2}}$$

This value is nearly the same for all pure metals at ordinary temperatures.

Physical Explanation

- In metals, free electrons move through a periodic lattice of positive ions.
- When an electric field is applied, electrons transport charge, giving rise to electrical conductivity.
- When a temperature gradient is applied, the same electrons transport thermal energy, giving rise to thermal conductivity.
- Since both processes depend on the same charge carriers and scattering mechanisms (mainly electron–phonon scattering), a direct proportionality exists between $K$and σ.

Derivation (Qualitative)

From free electron theory:

$$\sigma = \frac{ne^2\tau}{m}$$

$$K = \frac{1}{3}Cv^2\tau$$

where

- n= number of free electrons
- e= electronic charge
- τ= relaxation time
- m= electron mass
- C= electronic specific heat
- v= average electron velocity

Since both conductivities depend on the same relaxation time τ, their ratio becomes proportional to temperature, leading to the Wiedemann–Franz law.

Validity of the Law

The Wiedemann–Franz law is valid when:

1. Heat and charge transport are dominated by electrons.
2. Scattering mechanisms for heat and charge are identical.
3. Temperature is not extremely low.

Limitations

- Deviates at very low temperatures.
- Not valid for semiconductors and insulators.
- Deviations occur in alloys and impure metals.
- Quantum effects cause corrections at low temperatures.

Importance

1. Confirms that electrons are the main carriers of heat and charge in metals.
2. Supports the free electron model of metals.
3. Helps estimate thermal conductivity from electrical conductivity.

The Wiedemann–Franz law provides a clear and elegant connection between electrical and thermal conductivities in metals. By showing that both properties originate from the motion of free electrons in a crystal lattice, the law forms a cornerstone in the understanding of transport phenomena in solid-state physics.

Unit 3: Magnetic Properties of Solids

1. Introduction
2. Permeability
3. Susceptibility
4. Relation between Permeability and Susceptibility
5. Classification of Magnetic Materials
6. Langevin's Theory of diamagnetism
7. Weiss theory of Para magnetism
8. Curie-Weiss law
9. Weiss theory of ferromagnetism
10. B-H Curve
11. Hysteresis and energy loss
12. soft and hard magnets

3.1 Introduction

Magnetic properties of solids describe the manner in which materials respond to an applied magnetic field. These properties arise due to the **magnetic moments associated with electrons** present in atoms and ions of a solid. The study of magnetism in solids is an important branch of solid-state physics, as it explains the behavior of materials used in electrical machines, memory devices, transformers, and modern electronic applications.

In solids, electrons contribute to magnetism mainly through:

1. **Orbital motion** of electrons around the nucleus
2. **Spin motion** of electrons about their own axis

Each electron behaves like a tiny magnetic dipole. The overall magnetic behavior of a solid depends on how these individual magnetic moments interact with each other and with an external magnetic field.

Origin of Magnetism in Solids

In an atom, the magnetic moment arises due to:

- Orbital angular momentum of electrons
- Spin angular momentum of electrons

In solids, atoms are closely packed in a crystal lattice. The interaction between neighboring atoms, crystal field effects, and thermal motion determine whether the magnetic moments align, oppose, or remain randomly oriented.

Magnetization

When a magnetic field is applied to a solid, the magnetic dipoles may align partially or completely with the field. This produces **magnetization**, defined as the magnetic moment per unit volume of the material. The extent of magnetization depends on:

- Strength of the applied magnetic field
- Temperature
- Nature of the material

Classification of Magnetic Materials

Based on their response to an external magnetic field, solids are broadly classified into:

1. **Diamagnetic materials** – weakly repelled by a magnetic field
2. **Paramagnetic materials** – weakly attracted by a magnetic field
3. **Ferromagnetic materials** – strongly attracted and retain magnetism
4. **Antiferromagnetic materials** – magnetic moments align antiparallel
5. **Ferrimagnetic materials** – unequal antiparallel alignment of moments

Each type exhibits distinct magnetic behavior due to different arrangements of magnetic moments in the crystal lattice.

Role of Temperature

Temperature plays a crucial role in determining magnetic properties:

- Thermal agitation tends to randomize magnetic moments.
- At high temperatures, magnetic ordering may be destroyed.
- Certain materials show magnetic transitions at characteristic temperatures such as the **Curie temperature** or **Néel temperature**.

Importance of Studying Magnetic Properties

1. Helps in understanding fundamental electronic structure of solids.
2. Essential for designing magnetic storage devices.
3. Important in power generation and transmission systems.
4. Forms the basis for advanced technologies such as spintronics and magnetic sensors.

3.2 Permeability

Magnetic permeability is a fundamental property of materials that describes **how easily magnetic field lines can pass through the material**. In other words, permeability tells us **how well a material supports the formation of a magnetic field inside it**. It is one of the key parameters in understanding magnetic circuits, electromagnets, transformers, inductors, and all magnetic materials.

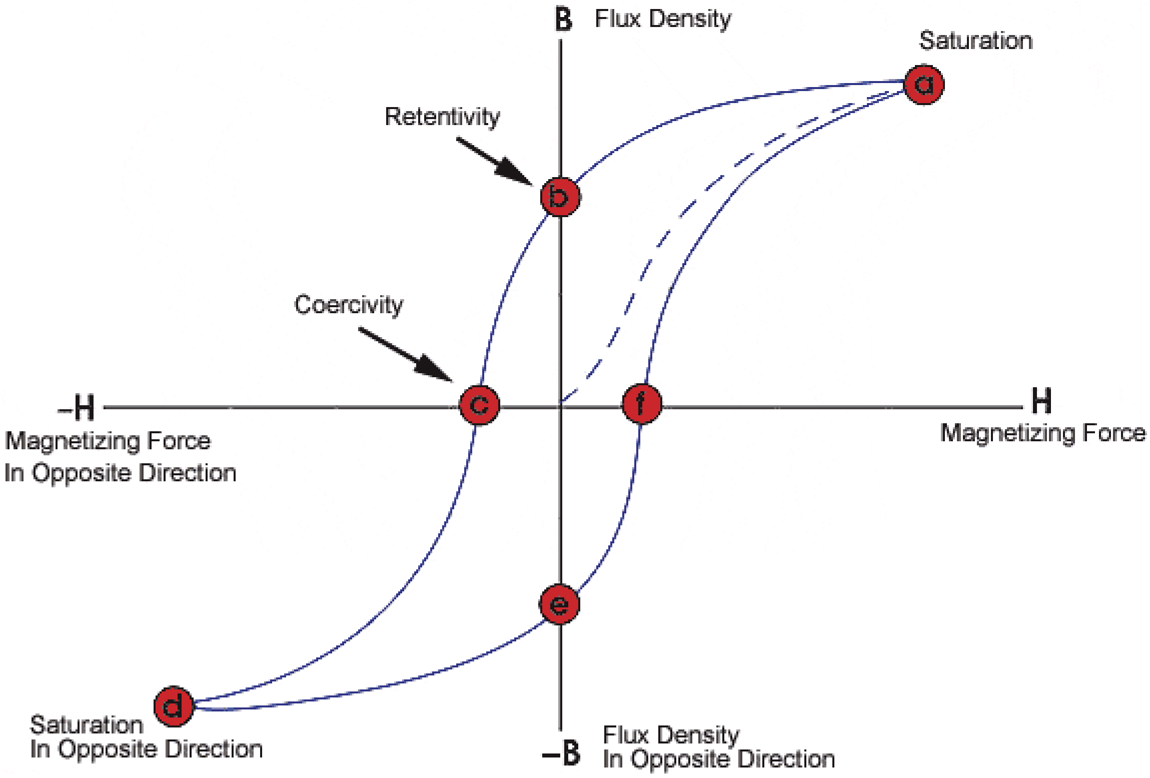


Figure 3. 1

Definition

Magnetic permeability (μ) is defined as:

$$\mu = \frac{B}{H}$$

where

B = Magnetic flux density (Tesla)

H = Magnetic field intensity (A/m)

Thus, permeability tells us how much magnetic flux density is produced inside a material when a magnetic field is applied.

Unit: Henry per meter (H/m)

Types of Permeability

1. Absolute Permeability (μ)

The actual permeability of any material.

$$B = \mu H$$

2. Permeability of Free Space (μ_0)

Also called **vacuum permeability**.

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

This is a constant value.

3. Relative Permeability (μ_r)

It compares the permeability of a material to free space.

$$\mu_r = \frac{\mu}{\mu_0}$$

No unit (dimensionless).

- If a material has **high permeability**, magnetic field lines easily pass through it.
- If a material has **low permeability**, magnetic field lines pass with difficulty.
- If permeability is **very high** (like in iron), the material becomes **strongly magnetised** even with small magnetic fields.

Thus, permeability indicates the **ease of magnetisation**.

Relation With Susceptibility

Magnetic susceptibility (χ) measures how easily a material gets magnetised.

Permeability is related to susceptibility as:

$$\mu = \mu_0(1 + \chi)$$

And

$$\mu_r = 1 + \chi$$

So,

• When χ is high → material has high permeability

• When χ is low → permeability is low

Permeability in Different Materials

1. Diamagnetic Materials

- $\mu_r < 1$
- Permeability slightly less than free space
- Weakly repelled by magnetic fields

  Examples: Copper, Bismuth

2. Paramagnetic Materials

- μ_r slightly > 1
- Weakly attracted

  Examples: Aluminium, Sodium

3. Ferromagnetic Materials

- $\mu_r >> 1$ (hundreds to thousands)
- Very strongly magnetised

  Examples: Iron, Nickel, Cobalt

Ferromagnetic materials are used in electromagnets and transformer cores because of their extremely high permeability.

Practical Importance

Magnetic permeability plays a key role in:

- Electromagnets
- Transformers
- Inductors
- Magnetic shielding
- Motors and generators
- Magnetic storage devices

Materials with high permeability concentrate magnetic flux and make devices more efficient.

3.3 Susceptibility

When a magnetic material is placed in an external magnetic field, its atoms or molecules try to align with the applied field. This alignment produces an additional internal magnetic moment within the material, known as **magnetisation (M)**.

Different materials respond differently to the same applied magnetic field.

Some materials are weakly magnetised, some strongly magnetised, and some even oppose the applied field.

To measure how easily or how strongly a material becomes magnetised due to an external field, we define a physical quantity called **magnetic susceptibility**, denoted by **χ**.

Magnetic susceptibility (χ) is defined as:

The ratio of magnetisation (M) produced in a material to the magnetic field intensity (H) applied to the material.

Mathematically,

$$\chi = \frac{M}{H}$$

or,

$$M = \chi H$$

Magnetic susceptibility is a dimensionless quantity.

- Magnetisation M represents the magnetic dipole moment per unit volume developed inside a material.
- Magnetic field intensity H represents the external magnetic field applied.
- Susceptibility χ tells how much magnetisation is produced for a given applied field.

Thus, susceptibility indicates **the degree of magnetisation of a material in response to an external magnetic field**.

Physical Interpretation

1. If a material has **high susceptibility**, it becomes **strongly magnetised** even under a small applied field.
2. If a material has **low susceptibility**, it becomes **weakly magnetised**.
3. If susceptibility is **negative**, the magnetisation is in the **opposite direction** to the applied field.

Thus, susceptibility gives information about:

- the nature of the material,
- the strength of magnetisation,
- whether the material is attracted or repelled by a magnetic field.

Types of Magnetic Susceptibility Based on Materials

Magnetic susceptibility varies depending on the type of material:

1. Diamagnetic Materials

- χ is **negative** and very small

$$\chi \approx -10^{-5} \text{ to } -10^{-6}$$

- These materials develop magnetisation in the **opposite direction** to the applied field.
- They are weakly repelled by magnetic fields.

Examples: Copper, Silver, Gold, Bismuth

2. Paramagnetic Materials

- χ is **positive** and small

$$\chi \approx +10^{-5} \text{ to } +10^{-3}$$

- They develop magnetisation in the **same direction** as the applied field.
- They are weakly attracted by magnetic fields.

Examples: Aluminium, Platinum, Sodium

3. Ferromagnetic Materials

- χ is **very large and positive**

$$\chi \gg 1$$

- They become **strongly magnetised** even under a small applied field.
- Magnetisation persists even after removing the external field.

Examples: Iron, Cobalt, Nickel

Relation with Permeability

Magnetic susceptibility is related to a material's relative permeability μ_r:

$$\mu_r = 1 + \chi$$

Thus, permeability of a material (how easily magnetic field can pass through it) depends directly on susceptibility.

Higher susceptibility → higher permeability.

Factors Affecting Magnetic Susceptibility

1. **Temperature**
   - For paramagnetic materials:

$$\chi \propto \frac{1}{T} \text{(Curie's Law)}$$

For ferromagnetic materials: susceptibility decreases as temperature increases.

2. **Nature of the material**
   - Atomic structure determines how easily dipoles align.
3. **Impurities**
   - Presence of other elements can increase or decrease susceptibility.

Significance of Magnetic Susceptibility

Magnetic susceptibility helps determine:

- how a material behaves in a magnetic field,

- whether magnetisation aligns or opposes the applied field,
- the classification of materials,
- design of magnetic devices like transformers, cores, inductors, memory devices, and sensors.

It is a fundamental property used in physics, chemistry, geophysics, and material science.

3.4 Relation between Permeability and Susceptibility

When a magnetic material is placed in an external magnetic field, it becomes magnetized. The extent to which a material responds to the applied magnetic field is described by two important magnetic parameters: **magnetic susceptibility** and **magnetic permeability**. These quantities are closely related and help in understanding the magnetic behavior of solids.

Magnetic Susceptibility (χ)

Magnetic susceptibility is a measure of how easily a material can be magnetized by an applied magnetic field.

It is defined as:

$$\chi = \frac{M}{H}$$

where

- M= intensity of magnetization
- H= magnetizing field intensity

Susceptibility is a **dimensionless quantity** and may be positive or negative depending on the nature of the material.

Magnetic Permeability (μ)

Magnetic permeability indicates the ability of a material to allow magnetic field lines to pass through it.

It is defined as:

$$\mu = \frac{B}{H}$$

where

- B= magnetic flux density
- H= magnetizing field intensity

The permeability of free space is denoted by μ_0.

Relation between B, H and M

In a magnetic material, the magnetic flux density $B$is given by:

$$B = \mu_0(H + M)$$

Substituting $M = \chi H$, we get:

$$B = \mu_0(H + \chi H)$$

$$B = \mu_0 H(1 + \chi)$$

Relation between Permeability and Susceptibility

Since $\mu = \frac{B}{H}$,

$$\mu = \mu_0(1 + \chi)$$

This is the required relation between magnetic permeability and magnetic susceptibility.

Relative Permeability

Relative permeability μ_r is defined as:

$$\mu_r = \frac{\mu}{\mu_0}$$

Substituting the value of μ,

$$\mu_r = 1 + \chi$$

Significance of the Relation

- For **diamagnetic materials**, χ is small and negative, so $\mu_r < 1$
- For **paramagnetic materials**, χ is small and positive, so $\mu_r > 1$
- For **ferromagnetic materials**, χ is very large, hence $\mu_r \gg 1$

This relation helps in classifying magnetic materials based on their magnetic response.

Magnetic permeability and magnetic susceptibility are directly related quantities that describe the magnetic behavior of materials. The simple relation $\mu = \mu_0(1 + \chi)$ connects microscopic magnetization with macroscopic magnetic field properties, forming a fundamental concept in the study of magnetic properties of solids.

3.5 Classification of Magnetic Materials

Magnetic materials are classified based on their response to an external magnetic field. This response depends on the nature of atomic magnetic moments, their interaction with each other, and their alignment in the crystal lattice. Based on magnetic susceptibility and permeability, solids are broadly classified into five main types: **diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic, and ferrimagnetic materials**.

1. Diamagnetic Materials

Characteristics

- Weakly repelled by a magnetic field
- Magnetic susceptibility is small and negative
- Relative permeability is slightly less than unity
- Magnetization is opposite to the applied field

Diamagnetism arises due to the induced orbital motion of electrons when an external magnetic field is applied. All electrons are paired, resulting in no permanent magnetic moment.

Examples

Bismuth, copper, silver, gold, silicon, water

2. Paramagnetic Materials

Characteristics

- Weakly attracted by a magnetic field
- Magnetic susceptibility is small and positive
- Relative permeability is slightly greater than unity
- Magnetization disappears when the field is removed

Paramagnetism occurs due to the presence of unpaired electrons. In the absence of a magnetic field, the magnetic moments are randomly oriented due to thermal motion.

Examples

Aluminium, platinum, chromium, manganese, oxygen

3. Ferromagnetic Materials

Characteristics

- Strongly attracted by a magnetic field
- Magnetic susceptibility is very large and positive
- Relative permeability is much greater than unity
- Retain magnetization even after the field is removed

In ferromagnetic materials, atomic magnetic moments align parallel to each other due to strong exchange interaction. The material is divided into regions called magnetic domains.

Curie Temperature

Above a certain temperature (Curie temperature), ferromagnetic materials lose their ferromagnetism and become paramagnetic.

Examples

Iron, cobalt, nickel, gadolinium

4. Antiferromagnetic Materials

Characteristics

- Weak magnetic behavior
- Magnetic moments align antiparallel with equal magnitude
- Net magnetic moment is zero

Due to exchange interaction, neighboring atomic moments align in opposite directions, canceling each other.

Néel Temperature

Above the Néel temperature, antiferromagnetic materials become paramagnetic.

Examples

Manganese oxide (MnO), nickel oxide (NiO), chromium oxide (Cr_2O_3)

5. Ferrimagnetic Materials

Characteristics

- Moderately strong magnetic behavior
- Magnetic moments align antiparallel but with unequal magnitudes
- Net magnetic moment is non-zero

Similar to antiferromagnetism, but unequal opposing moments lead to partial magnetization.

Examples

Magnetite (Fe_3O_4), ferrites such as $MgFe_2O_4$, $ZnFe_2O_4$

Comparison

Type	**Susceptibility (χ)**	**Permeability (μ_r)**	**Net Magnetization**
Diamagnetic	Negative	< 1	Very small
Paramagnetic	Positive (small)	> 1	Small
Ferromagnetic	Positive (large)	≫ 1	Very large
Antiferromagnetic	Small	≈ 1	Zero
Ferrimagnetic	Moderate	> 1	Finite

3.6 Langevin's Theory of diamagnetism

Langevin's theory provides a classical explanation for **diamagnetism**, which is exhibited by all materials to some extent. Diamagnetism arises due to the modification of the **orbital motion of electrons** in an atom when an external magnetic field is applied. According to this theory, diamagnetic substances develop a magnetic moment **opposite to the direction of the applied magnetic field**, resulting in weak repulsion.

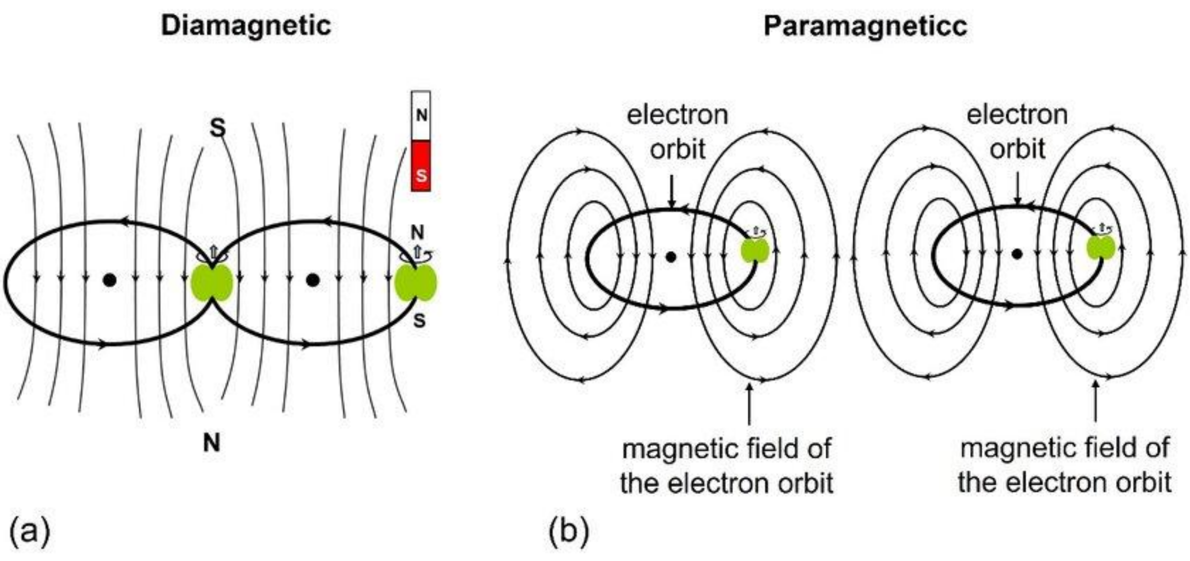


Figure 3. 2

Basic Assumptions of the Theory

1. Atoms contain electrons revolving in closed orbits around the nucleus.
2. In the absence of a magnetic field, the magnetic moments due to electron orbits cancel each other.
3. When an external magnetic field is applied, the orbital motion of electrons is modified.
4. The induced magnetic moment opposes the applied magnetic field.

Explanation of Diamagnetism

When a magnetic field **B**is applied perpendicular to the plane of an electron orbit:

- An additional force acts on the electron due to the magnetic field.
- This causes a change in the angular velocity of the electron.
- The change in angular velocity results in an **induced current** in the orbit.
- The induced current produces a magnetic moment opposite to the applied field.

This phenomenon is known as **Larmor precession**.

Induced Magnetic Moment

The induced magnetic moment μ due to an electron orbit is given by:

$$\mu = -\frac{e^2 r^2 B}{6m}$$

where

- e= charge of the electron
- r= radius of the electron orbit
- m= mass of the electron
- B= magnetic field

The negative sign indicates that the induced magnetic moment is opposite to the applied magnetic field.

Magnetic Susceptibility

The diamagnetic susceptibility χ_d is given by:

$$\chi_d = -\frac{\mu_0 N e^2 \langle r^2 \rangle}{6m}$$

where

- N= number of atoms per unit volume
- $\langle r^2 \rangle$= mean square radius of electron orbit
- μ_0= permeability of free space

Important Features of Diamagnetism

1. Diamagnetic susceptibility is **negative**.
2. Diamagnetism is independent of temperature.
3. Present in all materials but usually masked by stronger magnetic effects.
4. Does not require unpaired electrons.

Merits of Langevin's Theory

- Explains the origin of diamagnetism on classical grounds.
- Predicts negative susceptibility correctly.
- Explains temperature independence of diamagnetism.

Limitations

- Based on classical electron orbits, which are not fully accurate.
- Does not include quantum mechanical effects.
- Cannot explain anisotropic diamagnetism in some crystals.

Langevin's theory of diamagnetism explains diamagnetic behavior as a result of induced changes in the orbital motion of electrons when a magnetic field is applied. Although classical in nature, the theory successfully accounts for the weak, negative, and temperature-independent magnetic susceptibility observed in diamagnetic materials.

3.7 Weiss theory of Para magnetism

Weiss theory of paramagnetism is an extension of **Langevin's classical theory**, introduced to explain deviations from Curie's law observed in some paramagnetic substances. According to Weiss, in addition to the externally applied magnetic field, atoms in a magnetic material experience an **internal or molecular field** due to interactions with neighboring magnetic moments. This concept leads to the **Curie–Weiss law**, which successfully explains the temperature dependence of susceptibility in many paramagnetic materials.

Basic Assumptions of Weiss Theory

1. Paramagnetic substances consist of atoms or ions having permanent magnetic dipole moments.
2. Each magnetic moment experiences not only the applied magnetic field H but also an internal molecular field H_m.
3. The molecular field is proportional to the magnetization M of the material.
4. Thermal agitation tends to randomize the orientation of magnetic moments.

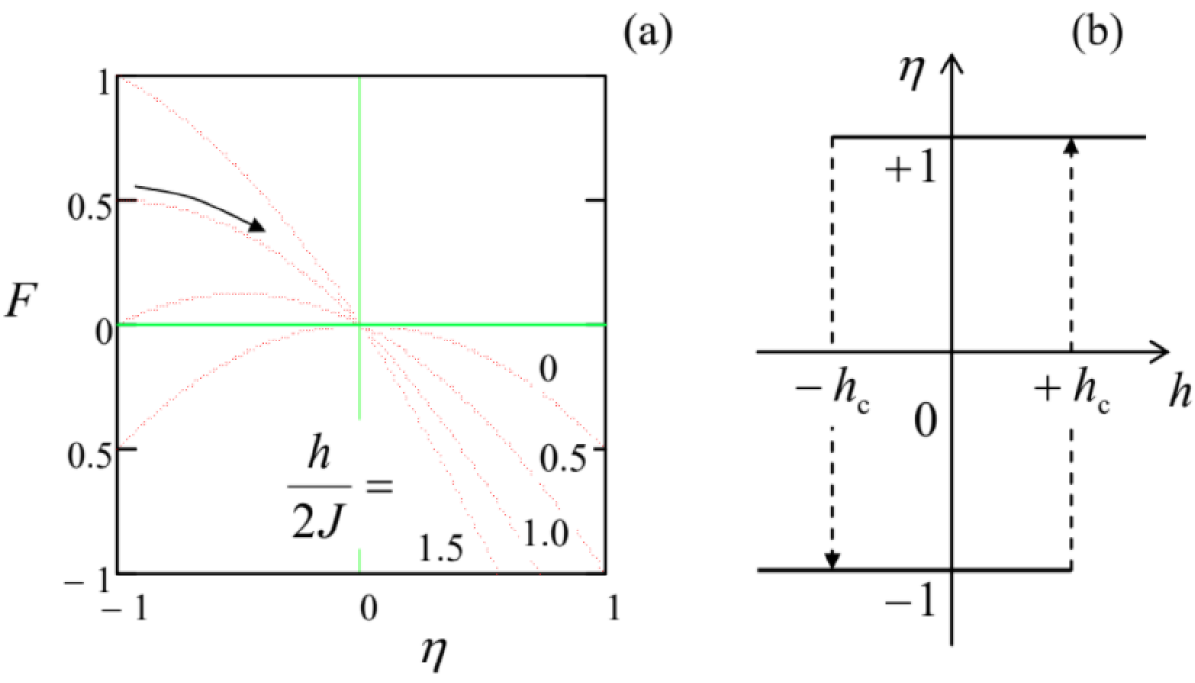


Figure 3. 3

Molecular Field Concept

Weiss proposed that the effective magnetic field acting on each atom is:

$$H_{\text{eff}} = H + H_m$$

The molecular field is given by:

$$H_m = \lambda M$$

where

- λ= Weiss constant
- M= magnetization

Thus,

$$H_{\text{eff}} = H + \lambda M$$

Magnetization in Paramagnetic Substances

For paramagnetic materials, magnetization is proportional to the effective field:

$$M = \chi H_{\text{eff}}$$

Substituting for H_{eff},

$$M = \chi(H + \lambda M)$$

Rearranging,

$$M(1 - \chi\lambda) = \chi H$$

Curie–Weiss Law

From the above relation, the magnetic susceptibility is obtained as:

$$\chi = \frac{C}{T - \theta}$$

where

- C= Curie constant
- T= absolute temperature

- θ= Weiss temperature

This is known as the **Curie–Weiss law**.

Significance of Weiss Temperature

- For **paramagnetic substances**, $\theta$is small and positive.
- When $T \gg \theta$, Curie–Weiss law reduces to Curie's law:

$$\chi = \frac{C}{T}$$

Important Features of Weiss Theory

1. Explains deviation from Curie's law at low temperatures.
2. Introduces the concept of internal molecular field.
3. Predicts linear variation of $1/\chi$with temperature.
4. Forms the basis for explaining ferromagnetism at lower temperatures.

Merits of Weiss Theory

- Provides better agreement with experimental results than Langevin theory.
- Explains temperature-dependent susceptibility accurately.
- Establishes a link between paramagnetism and ferromagnetism.

Limitations

- Molecular field is introduced phenomenologically without microscopic justification.
- Quantum mechanical effects are not included.
- Not accurate for all paramagnetic materials.

Weiss theory of paramagnetism improves upon classical theories by introducing the concept of a molecular field. It successfully explains the temperature dependence of magnetic susceptibility and leads to the Curie–Weiss law, making it a significant development in the study of magnetic properties of solids.

3.8 Curie-Weiss law

The Curie–Weiss law describes the temperature dependence of **magnetic susceptibility** of paramagnetic materials. It is an extension of **Curie's law** and was proposed by Pierre Weiss to explain the observed deviations from Curie's law at lower temperatures. This law introduces the concept of an internal **molecular field** acting within the material.

Statement of Curie–Weiss Law

The Curie–Weiss law states that the magnetic susceptibility χ of a paramagnetic substance is given by:

$$\chi = \frac{C}{T - \theta}$$

where

- χ= magnetic susceptibility
- C= Curie constant
- T= absolute temperature
- θ= Weiss constant or Weiss temperature

Physical Meaning of Weiss Temperature (θ)

- θ represents the strength of internal molecular interactions.

- It indicates the tendency of magnetic moments to align even in the absence of an external field.

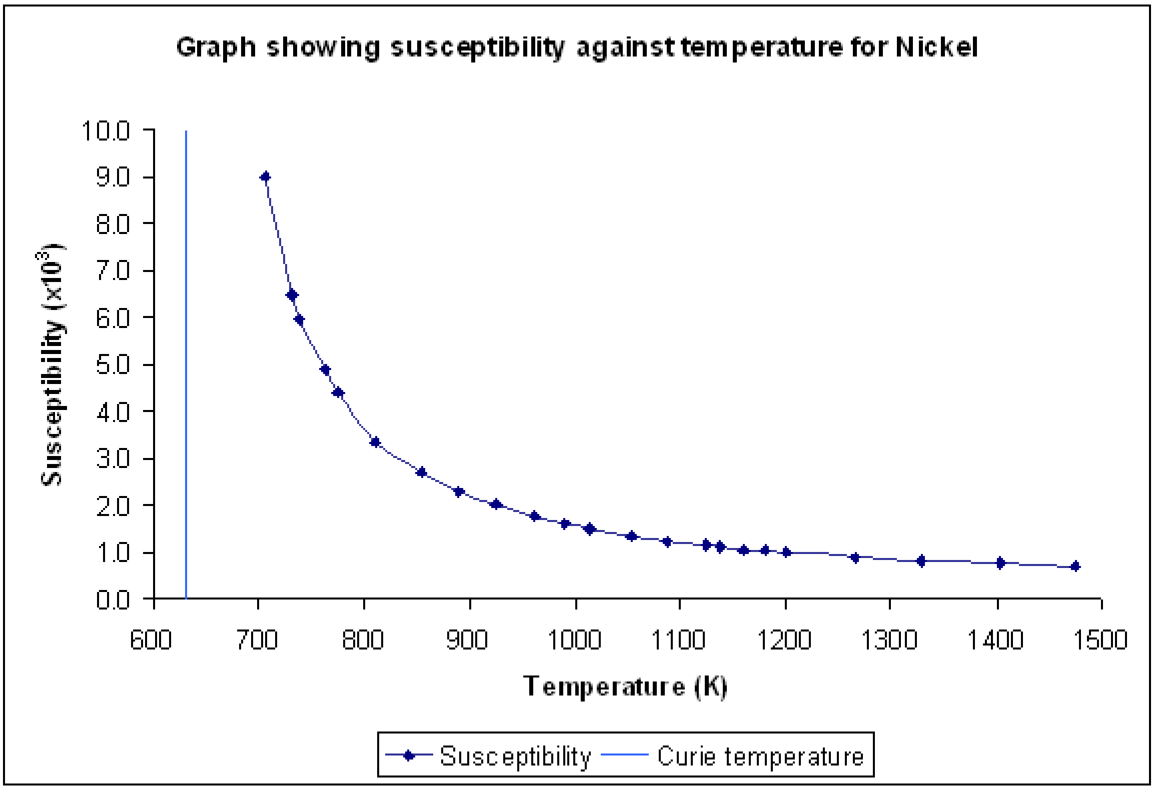


Figure 3. 4

- The value and sign of θprovide information about the magnetic nature of the material.

Special Cases

1. **High Temperature Region ($T \gg \theta$)**

   The Curie–Weiss law reduces to Curie's law:

$$\chi = \frac{C}{T}$$

2. **Ferromagnetic Materials**

   $\theta$is positive and close to the Curie temperature.

3. **Antiferromagnetic Materials**

   $\theta$is negative.

Graphical Representation

- A plot of χversus Tshows a rapid increase in susceptibility as temperature approaches θ.
- A plot of $\frac{1}{\chi}$versus $T$is a straight line.
- The intercept on the temperature axis gives the value of θ.

Importance of Curie–Weiss Law

1. Explains deviation from Curie's law at low temperatures.
2. Helps identify magnetic phase transitions.
3. Distinguishes between paramagnetic, ferromagnetic, and antiferromagnetic materials.
4. Provides experimental determination of Weiss temperature.

Limitations

- Valid only above the magnetic ordering temperature.
- Does not accurately describe behaviour at very low temperatures.
- Based on phenomenological molecular field assumptions.

The Curie–Weiss law provides a more accurate description of paramagnetic behavior than Curie's law by accounting for internal molecular interactions. It plays a crucial role in understanding magnetic ordering and phase transitions in solid-state physics.

3.9 Weiss theory of ferromagnetism

Weiss theory of ferromagnetism was proposed to explain the strong magnetic behavior exhibited by ferromagnetic materials such as iron, cobalt, and nickel. Unlike paramagnetic substances, ferromagnetic materials show **spontaneous magnetization**, that is, they remain magnetized even in the absence of an external magnetic field. Weiss explained this behavior by introducing the concept of a strong **internal molecular field**.

Basic Idea of Weiss Theory

According to Weiss, each atom in a ferromagnetic material possesses a permanent magnetic moment due to electron spin. In addition to the externally applied magnetic field, each magnetic moment experiences a very strong **molecular field** produced by neighboring atoms. This internal field tends to align all magnetic moments in the same direction.

The molecular field is assumed to be proportional to the magnetization of the material:

$$H_m = \lambda M$$

where

- H_m is the molecular field
- λ is the Weiss constant
- M is the magnetization

Spontaneous Magnetization

Even when no external magnetic field is applied:

- The molecular field is strong enough to align atomic magnetic moments parallel to each other.
- As a result, the material shows **spontaneous magnetization**.
- This is the defining characteristic of ferromagnetic materials.

Magnetic Domains

Weiss introduced the concept of **magnetic domains** to explain why an unmagnetized ferromagnetic specimen may show zero net magnetization.

- A ferromagnetic crystal is divided into small regions called domains.
- Within each domain, magnetic moments are perfectly aligned.
- In the absence of an external field, domains are randomly oriented, giving zero net magnetization.
- When a magnetic field is applied, domains aligned with the field grow at the expense of others, producing strong magnetization.

Concept of Domains

Weiss explained that a ferromagnetic crystal is divided into many small regions called **magnetic domains**.

- Within each domain, atomic magnetic moments are perfectly aligned in the same direction.
- Each domain is uniformly magnetized to saturation.
- Adjacent domains are separated by thin transition regions called **domain walls** or **Bloch walls**.

Unmagnetized Ferromagnetic Specimen

- In the absence of an external magnetic field, domains are oriented randomly.
- The magnetizations of different domains cancel each other.
- Hence, the specimen shows **zero net magnetization**, even though each domain is magnetized.

Magnetization Process

When an external magnetic field is applied:

1. Domains oriented parallel to the field grow in size.
2. Domain walls move, increasing the volume of favorably oriented domains.
3. At higher fields, magnetic moments within domains rotate toward the field direction.
4. Eventually, all domains align, producing **saturation magnetization**.

Domain Wall Motion

- Domain walls move easily under weak magnetic fields.
- Impurities, defects, and lattice imperfections can hinder wall motion.
- This resistance to domain wall motion leads to **hysteresis**.

Importance of Domain Concept

1. Explains why ferromagnetic materials can be magnetized strongly.
2. Accounts for zero magnetization in unmagnetized specimens.
3. Helps understand hysteresis, coercivity, and remanence.
4. Forms the basis for magnetic storage and transformer core design.

Effect of Temperature

Thermal agitation tends to disturb the alignment of magnetic moments.

- At low temperatures, molecular field dominates, and ferromagnetism is observed.
- As temperature increases, thermal energy opposes alignment.
- At a certain temperature called the **Curie temperature**, spontaneous magnetization disappears.
- Above the Curie temperature, the material behaves as a paramagnetic substance.

Explanation of Curie Temperature

At the Curie temperature:

- Thermal energy becomes comparable to the molecular field energy.
- Long-range magnetic order is destroyed.
- Ferromagnetic material transitions to the paramagnetic state.

Merits of Weiss Theory

1. Explains spontaneous magnetization in ferromagnetic materials.
2. Introduces the concept of molecular field and magnetic domains.
3. Explains the existence of Curie temperature.
4. Provides a qualitative understanding of ferromagnetic behavior.

Limitations

1. Molecular field is introduced without microscopic justification.
2. Does not explain the origin of domain structure quantitatively.
3. Quantum mechanical exchange interaction is not explicitly included.

Weiss theory of ferromagnetism qualitatively explains the strong magnetic properties of ferromagnetic materials by introducing the concept of an internal molecular field and magnetic domains. Although phenomenological in nature, the theory successfully accounts for spontaneous magnetization, domain formation, and the disappearance of ferromagnetism above the Curie temperature, forming a foundation for modern theories of magnetism.

3.10 B-H Curve

B–H curve (Magnetisation Curve) shows how a magnetic material responds when an external magnetising field **H** is applied.

It helps us study:

- Magnetisation behaviour
- Saturation
- Retentivity
- Coercivity
- Hysteresis

To obtain this curve experimentally, a **toroidal ring specimen** with coils wound on it is used.

To **plot the B–H curve** of a ferromagnetic specimen by measuring:

- **Magnetic field intensity (H)** using the magnetising coil
- **Magnetic flux density (B)** using the search coil and ballistic galvanometer

Apparatus Required

- Toroidal iron ring specimen
- Magnetising coil (primary coil)
- Search coil (secondary coil)
- Ballistic galvanometer
- Rheostat
- Ammeter

- Voltmeter
- Keys and connecting wires
- DC regulated supply

Circuit Diagram

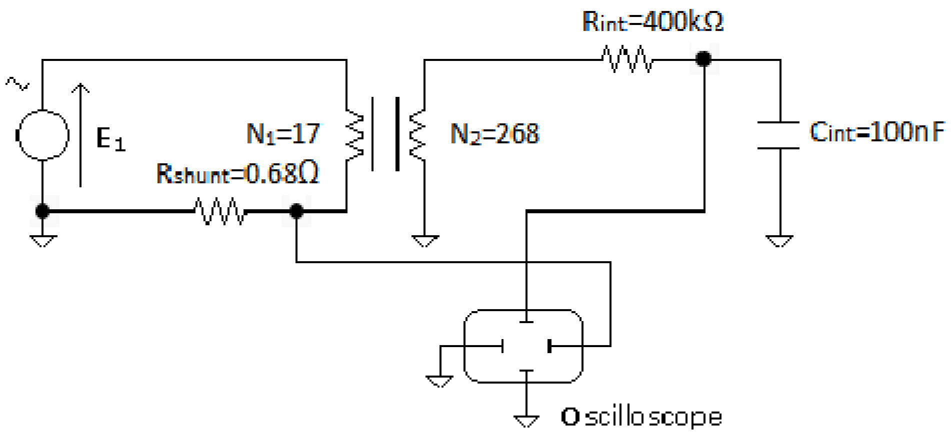


Figure 3. 5

Theory

For a toroidal core:

Magnetising Field (H)

$$H = \frac{NI}{l}$$

where

N = number of turns in magnetising coil

I = current in magnetising coil

l = mean magnetic path length of the toroid

Magnetic Flux Density (B)

Flux change induces a galvanometer deflection:

$$B = k\,\theta$$

where

θ = galvanometer deflection

k = calibration constant for search coil + galvanometer system

Thus, for different values of **I**, you find corresponding **H** and **B**, and plot **B vs H**.

Procedure

1. Initial Setup

- Connect the magnetising coil in series with ammeter, rheostat, DC supply.
- Connect the search coil to the ballistic galvanometer.
- Ensure connections are tight and polarity correct.

2. Increasing Magnetising Current

- Slowly increase the current in steps using the rheostat.
- At each current value:
    - Note the ammeter reading (to calculate **H**)
    - Momentarily press the search coil key → galvanometer deflects
    - Record the deflection (to calculate **B**)

3. Obtaining the B–H Curve

- Continue increasing current until the material **reaches saturation**.
- Plot B against H → gives the **magnetisation curve**.

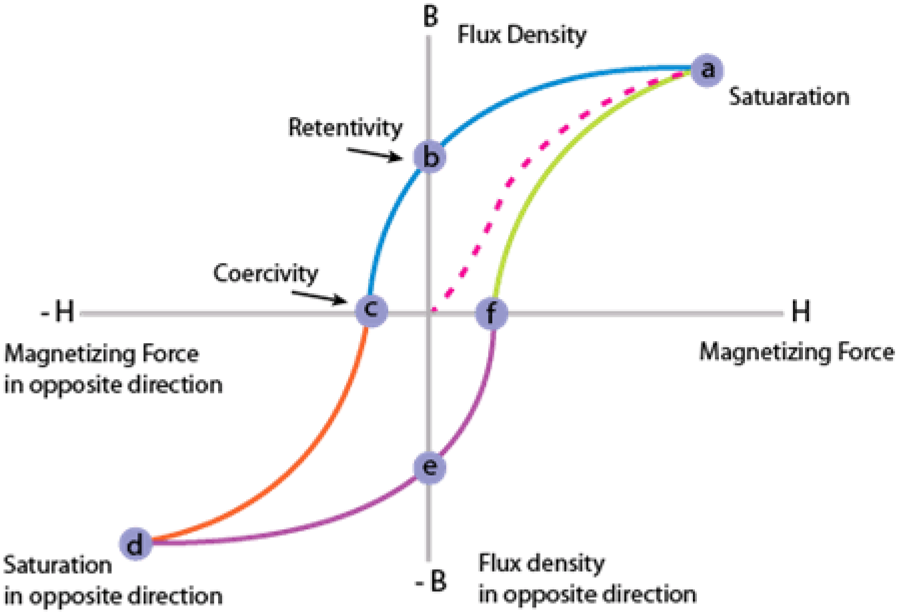


Figure 3. 6

4. Obtaining the Hysteresis Loop

- After saturation, gradually **reduce** the current back to zero.
- Now make current flow in the **reverse direction**.
- Note deflection values during decreasing and reversing of current.
- Continue until saturation in the opposite direction is reached.
- Plot entire cycle — this gives the **hysteresis loop**.

The curve shows:

- **O–A:** Initial magnetisation
- **A–B:** Saturation
- **C:** Residual magnetism (Retentivity)
- **D:** Coercive force (Coercivity)

Observations:

Current (I)	Magnetising Field (H)	Deflection (θ)	Flux Density (B)
…	…	…	…

Result

1. **The B–H curve** of the given ferromagnetic specimen was successfully plotted.
2. The material shows **saturation**, **retentivity**, and **coercivity**, typical of ferromagnets.
3. From the hysteresis loop, energy loss per cycle can also be estimated.

3.11 Hysteresis and energy loss

When a magnetic material (like iron) is taken through a complete cycle of magnetization — from zero field to positive saturation, back through zero, then to negative saturation, and again to zero — the **magnetic induction (B)** does **not** follow the same path as the magnetizing field **H**.This lag between **B** and **H** forms a closed loop called the **hysteresis loop**.

Because of this lag, some energy is **lost** in every cycle of magnetization. This loss appears as **heat** inside the magnetic material. This is known as **hysteresis loss**.

Why Does Energy Get Lost?

A magnetic material consists of many tiny magnetic domains.

During magnetization:

- Domains **rotate**
- Domain walls **move**
- Some domains are **reluctant to align** with the external field
- Internal friction inside the material **resists domain motion**

Every time H changes direction, the domains must rearrange again.

This rearrangement is not perfectly reversible — it **consumes energy**.

This consumed energy becomes **heat** inside the core.

The energy lost per cycle of magnetization is equal to the area enclosed by the hysteresis loop on the B–H curve.

Mathematically,

$$\text{Energy loss per unit volume per cycle} = \oint H\, dB$$

This integral represents the **area of the hysteresis loop**.

How the Energy is Manifested

- The core gets **hot** during AC operation
- Heat must be removed to prevent damage
- Transformers and motors use materials with **narrow hysteresis loops** to reduce heating

Hysteresis Loss Formula

If an AC magnetic field of frequency f cycles per second is applied, the total loss per second is:

$$P_h = \eta\, B_{max}^{1.6}\, f\, V$$

Where:

- P_h= hysteresis power loss
- η= Steinmetz constant (depends on the material)
- B_{max}= maximum flux density
- f= frequency
- V= volume of the material

Key Points

- **Hysteresis loss = heat generated due to repeated magnetization and demagnetization.**
- It depends on **area of hysteresis loop**.

- Soft iron has **narrow loop → low hysteresis loss** (used in transformer cores).
- Hard steel has **wide loop → high hysteresis loss** (used in permanent magnets).
- Loss increases with **frequency** and **maximum flux density**.

3.12 Soft And Hard Magnets

Magnetic materials are classified as **soft magnets** and **hard magnets** based on how easily they can be magnetized and demagnetized. This classification is mainly determined by their **hysteresis behavior**, coercivity, retentivity, and domain wall motion. Soft and hard magnetic materials play distinct roles in electrical, electronic, and magnetic applications.

1. Soft Magnetic Materials

Soft magnetic materials are those which can be **easily magnetized and easily demagnetized**.

Characteristics

- Low coercivity
- Low retentivity
- High permeability
- Narrow hysteresis loop
- Low hysteresis loss
- Domain walls move easily

Magnetic Behavior

- When an external magnetic field is applied, domains align quickly.
- When the field is removed, magnetization almost completely disappears.
- Energy loss during magnetization cycles is very small.

Examples

Soft iron, silicon steel, permalloy, ferrites (soft ferrites)

Applications

- Transformer cores
- Electric motors and generators
- Electromagnets
- Relays and inductors

2. Hard Magnetic Materials

Definition

Hard magnetic materials are those which are **difficult to magnetize but retain magnetism strongly** once magnetized.

Characteristics

- High coercivity
- High retentivity
- Low permeability
- Wide hysteresis loop
- Large hysteresis loss
- Domain wall motion is strongly hindered

Magnetic Behavior

- Require a strong magnetic field for magnetization.
- Once magnetized, they do not lose magnetism easily.
- Suitable for permanent magnet applications.

Examples

Steel, Alnico, cobalt steel, hard ferrites, rare-earth magnets (Nd–Fe–B, Sm–Co)

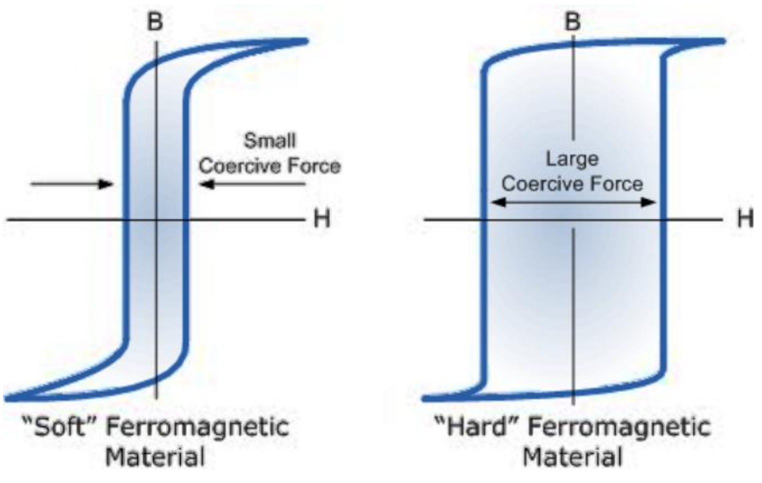


Figure 3. 7

Applications

- Permanent magnets
- Loudspeakers
- Magnetic recording devices
- Electric meters and sensors

Comparison between Soft and Hard Magnets

Property	Soft Magnets	Hard Magnets
Coercivity	Low	High
Retentivity	Low	High
Permeability	High	Low
Hysteresis loop	Narrow	Wide
Energy loss	Small	Large
Domain wall motion	Easy	Difficult
Typical use	Temporary magnets	Permanent magnets

Role of Domains

- In soft magnets, domain walls move freely due to fewer lattice imperfections.
- In hard magnets, impurities and defects pin domain walls, preventing easy realignment.

Soft and hard magnetic materials differ fundamentally in their response to magnetic fields. Soft magnets are suitable for applications requiring rapid and repeated magnetization and demagnetization with minimal energy loss, whereas hard magnets are ideal for permanent magnet applications due to their high retentivity and coercivity. Understanding this distinction is essential in the design and selection of magnetic materials for technological applications.

Unit 4: Dielectric Properties of Materials

1. Introduction
2. Basic Definition of Polarization and electric susceptibility
3. Local electric field of an atom
4. Dielectric constant and polarizability
5. Polarization Processes
6. Electronic Polarization
7. Calculation of polarizability
8. Ionic, Orientational and space charge polarization
9. Internal field
10. Clausius-Mosotti relation
11. Frequency dependence of dielectric constant
12. Dielectric loss
13. effect of temperature on dielectric constant

4.1 Introduction

Dielectric properties of materials describe their behavior when subjected to an external electric field. Dielectrics are insulating materials that do not conduct electric current but become **polarized** in the presence of an electric field. The study of dielectric properties is important in solid-state physics because it explains energy storage, polarization mechanisms, and the response of materials in capacitors, insulators, and electronic devices.

Dielectric Polarization

When an electric field is applied to a dielectric material:

- Positive and negative charges are displaced slightly.
- Electric dipoles are induced or aligned.
- The material develops an internal electric field opposite to the applied field.

This phenomenon is called **dielectric polarization**.

Types of Polarization

Dielectric polarization occurs due to different mechanisms:

1. **Electronic Polarization**
   - Caused by displacement of electron cloud relative to the nucleus.
   - Occurs in all dielectric materials.
   - Dominant at high frequencies.
2. **Ionic Polarization**
   - Occurs in ionic solids due to relative displacement of positive and negative ions.
   - Depends on the nature of the ionic bond.
3. **Orientational (Dipolar) Polarization**
   - Arises in materials having permanent dipole moments.
   - Dipoles align with the applied field.
   - Strongly temperature dependent.
4. **Space Charge Polarization**
   - Due to accumulation of charges at grain boundaries or interfaces.

   - Occurs at low frequencies.

Dielectric Constant

The dielectric constant (relative permittivity) is a measure of the ability of a material to store electrical energy.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

where

- ε= permittivity of the material
- ε_0= permittivity of free space

A higher dielectric constant indicates greater polarization.

Dielectric Susceptibility

Dielectric susceptibility $\chi_e$relates polarization $P$to the applied electric field E:

$$P = \varepsilon_0 \chi_e E$$

The relation between dielectric constant and susceptibility is:

$$\varepsilon_r = 1 + \chi_e$$

Frequency Dependence of Dielectric Constant

- At low frequencies, all polarization mechanisms contribute.
- As frequency increases, dipolar and space charge polarization fail to follow the field.
- At very high frequencies, only electronic polarization remains.
- Hence, dielectric constant decreases with increasing frequency.

Dielectric Loss

When an alternating electric field is applied:

- Part of the electrical energy is dissipated as heat.
- This energy loss is called **dielectric loss**.

It is represented by the **loss tangent**:

$$\tan\delta = \frac{\varepsilon''}{\varepsilon'}$$

where

- ε'= dielectric constant
- ε''= dielectric loss factor

Applications of Dielectric Materials

1. Capacitors for energy storage
2. Electrical insulation
3. Microwave and RF devices
4. Sensors and transducers
5. Solid-state electronic components

Dielectric properties of materials arise from the polarization of charges under an applied electric field. Different polarization mechanisms contribute depending on frequency and temperature. Understanding dielectric behavior is essential for the design and application of insulating and energy-storage materials in modern electronic and electrical systems.

4.2 Basic Definition of Polarization and electric susceptibility

When a dielectric material is placed in an external electric field, the bound charges within the atoms or molecules undergo slight displacement. As a result, electric dipoles are formed or aligned inside the material. This phenomenon is known as **polarization**. The extent to which a dielectric material gets polarized under an applied electric field is measured by a quantity called **electric susceptibility**. These two concepts are fundamental to understanding the dielectric behavior of materials.

Electric Polarization

Definition

Electric polarization is defined as the **electric dipole moment per unit volume** of a dielectric material.

$$\vec{P} = \frac{\text{Total electric dipole moment}}{\text{Volume}}$$

where

- $\vec{P}$= polarization vector

The direction of polarization is the same as that of the applied electric field.

Physical Origin of Polarization

Polarization occurs due to the displacement of charges inside the dielectric when an electric field is applied. Depending on the nature of the material, polarization arises through different mechanisms such as electronic, ionic, orientational, and space charge polarization.

Polarization Vector

The polarization vector $\vec{P}$ represents:

- The strength of polarization
- The direction of alignment of dipoles

It gives a macroscopic description of how dipoles are distributed inside the dielectric.

Electric Susceptibility

Definition

Electric susceptibility χ_e is a measure of how easily a dielectric material becomes polarized when subjected to an electric field.

It is defined as:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

where

- ε_0= permittivity of free space
- $\vec{E}$= applied electric field
- χ_e= electric susceptibility

Electric susceptibility is a **dimensionless quantity**.

Relation between Polarization and Electric Field

From the above relation:

- Greater the value of χ_e, greater is the polarization produced for a given electric field.
- Materials with large susceptibility polarize more easily.

Relation between Dielectric Constant and Susceptibility

The dielectric constant (relative permittivity) ε_r of a material is related to electric susceptibility by:

$$\varepsilon_r = 1 + \chi_e$$

This relation connects the microscopic polarization of dipoles with the macroscopic dielectric behavior of the material.

Important Features

- Polarization exists only when an external electric field is applied.
- Electric susceptibility depends on the nature of the material.
- For vacuum, $\chi_e = 0$.
- For dielectrics, $\chi_e > 0$.

Temperature and Frequency Dependence

- Orientational polarization decreases with increase in temperature.
- At high frequencies, dipoles cannot follow the rapidly varying electric field, reducing polarization.
- Hence, electric susceptibility decreases with increasing frequency.

Polarization describes the formation and alignment of electric dipoles in a dielectric material under an applied electric field, while electric susceptibility quantifies the ease with which this polarization occurs. Together, these concepts provide a fundamental understanding of dielectric behavior and form the basis for the study of capacitors, insulators, and dielectric materials in solid-state physics.

4.3 Local electric field of an atom

When a dielectric material is placed in an external electric field, each atom or molecule inside the dielectric does not experience the applied field alone. Instead, it is subjected to an **effective field**, known as the **local electric field**. The local electric field plays a crucial role in understanding polarization, dielectric constant, and microscopic electric behavior of materials.

Need for Local Electric Field

The macroscopic electric field E inside a dielectric represents an average field over a large number of atoms. However, polarization is produced at the **atomic or molecular level**. Therefore, to explain polarization accurately, it is necessary to determine the actual electric field acting on an individual atom, which is the local electric field.

Definition

The **local electric field** is defined as the total electric field acting on a particular atom or molecule inside a dielectric material. It is different from the macroscopic field due to contributions from nearby charges and dipoles.

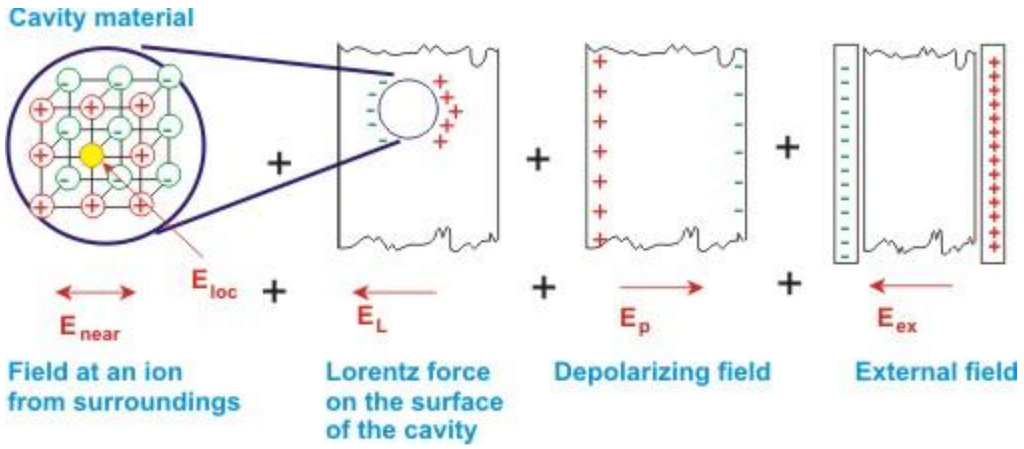


Figure 4. 1

Contributions to the Local Electric Field

The local electric field E_{local}at an atom consists of the following components:

1. **Applied (Macroscopic) Electric Field** E

   This is the externally applied electric field acting on the dielectric.

2. **Field Due to Polarization Charges on the Surface**

   When the dielectric is polarized, bound charges appear on its surfaces. These charges produce an additional electric field inside the material.

3. **Field Due to Dipoles Inside the Dielectric**

   Neighboring polarized atoms act as dipoles and contribute to the local field.

4. **Lorentz Field**

   The most important contribution arising from dipoles within a small spherical cavity around the atom.

Lorentz Cavity Concept

To calculate the local electric field, Lorentz imagined a small **spherical cavity** around the chosen atom inside the dielectric:

- All dipoles inside the cavity are removed.
- The field inside the cavity due to polarization of the surrounding medium is calculated.
- The atom at the center experiences this additional field.

Lorentz Local Field

For an isotropic dielectric, the local electric field is given by:

$$E_{\text{local}} = E + \frac{P}{3\varepsilon_0}$$

where

- E= macroscopic electric field
- P= polarization of the dielectric
- ε_0= permittivity of free space

The term $\frac{P}{3\varepsilon_0}$is called the **Lorentz field**.

Significance of Local Electric Field

- Determines the polarization produced by individual atoms.
- Helps derive the **Clausius–Mossotti relation**.
- Explains the microscopic origin of dielectric constant.

- Important in optical and dielectric response of materials.

Assumptions in Lorentz Theory

1. The dielectric is homogeneous and isotropic.
2. Atoms are arranged symmetrically.
3. The cavity is spherical.
4. Short-range interactions are neglected.

Limitations

- Not valid for anisotropic crystals.
- Assumes uniform polarization.
- Neglects detailed atomic interactions.

The local electric field represents the actual electric field experienced by an atom inside a dielectric material. By introducing the Lorentz cavity and accounting for polarization effects, the concept of local electric field bridges the gap between macroscopic electric fields and microscopic atomic behavior. It forms a fundamental basis for understanding dielectric polarization and related solid-state phenomena.

4.4 Dielectric constant and polarizability

When a dielectric material is placed in an external electric field, it becomes polarized due to the displacement of bound charges within its atoms or molecules. The macroscopic response of a dielectric is described by the **dielectric constant**, while the microscopic response of individual atoms or molecules is described by **polarizability**. These two quantities ar closely related and form the basis for understanding dielectric behavior of materials.

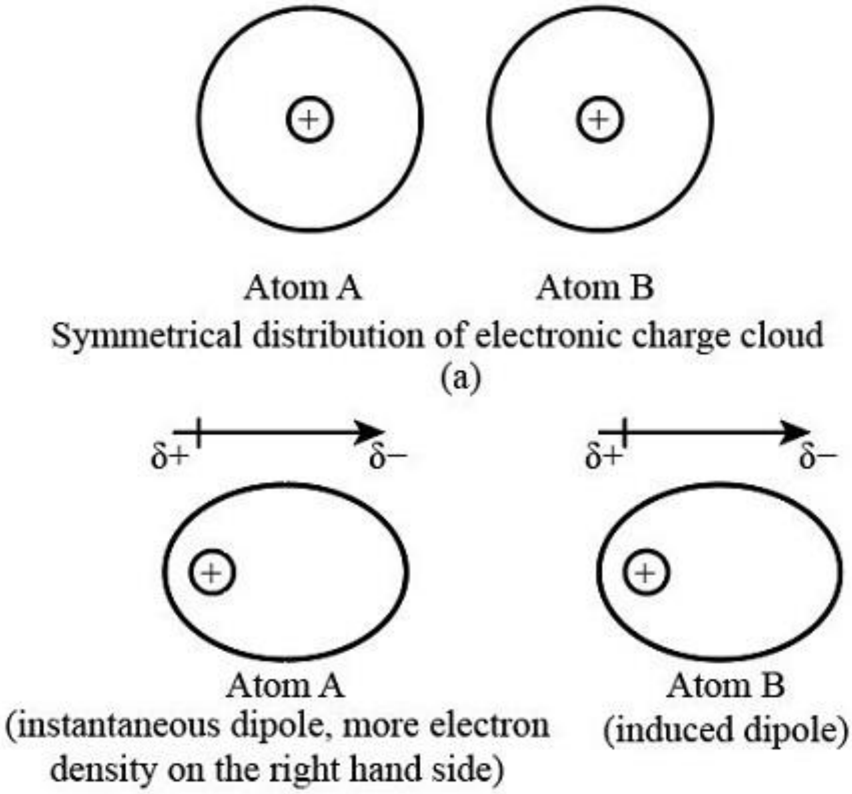


Figure 4. 2

Dielectric Constant (Relative Permittivity)

Definition

The dielectric constant of a material is defined as the ratio of the permittivity of the material to the permittivity of free space.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

where

- ε_r= dielectric constant
- ε= permittivity of the dielectric
- ε_0= permittivity of free space

Physical Significance

- Dielectric constant indicates the ability of a material to store electrical energy.
- A higher dielectric constant implies greater polarization for a given electric field.
- It determines the capacitance of a capacitor filled with the dielectric.

Relation with Electric Field

The electric displacement vector is given by:

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_0\varepsilon_r\vec{E}$$

Polarizability

Polarizability $\alpha$is defined as the **electric dipole moment induced per atom or molecule per unit applied electric field**.

$$p = \alpha E_{\text{local}}$$

where

- p= induced dipole moment
- α= polarizability
- E_{local}= local electric field acting on the atom

Types of Polarizability

1. **Electronic Polarizability** – due to displacement of electron cloud relative to nucleus
2. **Ionic Polarizability** – due to relative displacement of positive and negative ions
3. **Orientational Polarizability** – due to alignment of permanent dipoles
4. **Space Charge Polarizability** – due to accumulation of charges at interfaces

The total polarizability is the sum of all contributing mechanisms.

Relation between Polarization and Polarizability

Polarization $P$is the dipole moment per unit volume:

$$P = Np = N\alpha E_{\text{local}}$$

where

- N= number of atoms or molecules per unit volume

Using the Lorentz local field approximation:

$$E_{\text{local}} = E + \frac{P}{3\varepsilon_0}$$

Clausius–Mossotti Relation

Combining macroscopic and microscopic descriptions, the relation between dielectric constant and polarizability is obtained as:

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

This is known as the **Clausius–Mossotti relation**.

Significance of Clausius–Mossotti Relation

- Connects microscopic polarizability with macroscopic dielectric constant.
- Helps determine polarizability from experimental dielectric data.
- Valid for isotropic and non-polar dielectrics.

Important Features

- Dielectric constant depends on frequency and temperature.
- Polarizability depends on atomic structure and bonding.
- At high frequencies, only electronic polarizability contributes.

Limitations

- Not accurate for polar dielectrics.
- Assumes uniform and isotropic medium.
- Neglects strong intermolecular interactions.

The dielectric constant represents the overall response of a material to an applied electric field, while polarizability describes the response of individual atoms or molecules. The Clausius–Mossotti relation provides a vital link between these two quantities, offering a unified understanding of dielectric behavior from microscopic and macroscopic perspectives.

4.5 Polarization Processes

When a dielectric material is placed in an external electric field, the bound charges within the atoms or molecules undergo slight displacement. As a result, electric dipoles are either **induced** or **aligned**, producing polarization in the material. The sequence of physical events by which a dielectric becomes polarized under an applied electric field is known as the

polarization process. This process explains how dielectrics respond to electric fields at the microscopic and macroscopic levels.

Meaning of Polarization Process

The polarization process refers to the gradual development of polarization in a dielectric due to the action of an external electric field. It involves the rearrangement of charges or dipoles until an equilibrium state is reached between the applied electric field and opposing internal restoring forces.

Stages of Polarization

When an electric field is applied to a dielectric:

1. **Application of Electric Field**

   An external electric field is applied across the dielectric material.

2. **Displacement or Alignment of Charges**

   Positive and negative charges shift slightly in opposite directions, or permanent dipoles tend to align along the field direction.

3. **Formation of Dipoles**

   Electric dipoles are created (in non-polar materials) or aligned (in polar materials).

4. **Establishment of Internal Field**

   The polarized charges produce an internal electric field opposite to the applied field.

5. **Equilibrium State**

   Polarization reaches a steady value when electric forces balance restoring forces.

Mechanisms Involved in the Polarization Process

The polarization process occurs through different mechanisms depending on the nature of the dielectric:

1. Electronic Polarization

- Occurs due to displacement of the electron cloud relative to the nucleus.
- Takes place almost instantaneously.
- Present in all dielectric materials.
- Dominant at high frequencies.

2. Ionic Polarization

- Occurs in ionic solids due to relative displacement of positive and negative ions.
- Depends on the strength of ionic bonds.
- Slower than electronic polarization.

3. Orientational (Dipolar) Polarization

- Occurs in polar molecules with permanent dipole moments.
- Dipoles rotate and align with the applied field.
- Strongly dependent on temperature.

4. Space Charge Polarization

- Due to accumulation of charges at interfaces, grain boundaries, or defects.
- Occurs at low frequencies.
- Very slow polarization process.

Time Dependence of Polarization

- Polarization does not occur instantaneously in all materials.
- Some polarization mechanisms require finite time to respond.
- When the field is removed, polarization gradually decreases (dielectric relaxation).

Frequency Dependence

- At low frequencies, all polarization mechanisms contribute.
- With increasing frequency, slower mechanisms fail to follow the field.
- At very high frequencies, only electronic polarization remains.
- Hence, polarization decreases with increasing frequency.

Polarization and Dielectric Loss

During alternating electric fields:

- Delay in polarization response causes energy dissipation.
- This loss appears as heat and is called dielectric loss.
- It is significant in materials with orientational and space charge polarization.

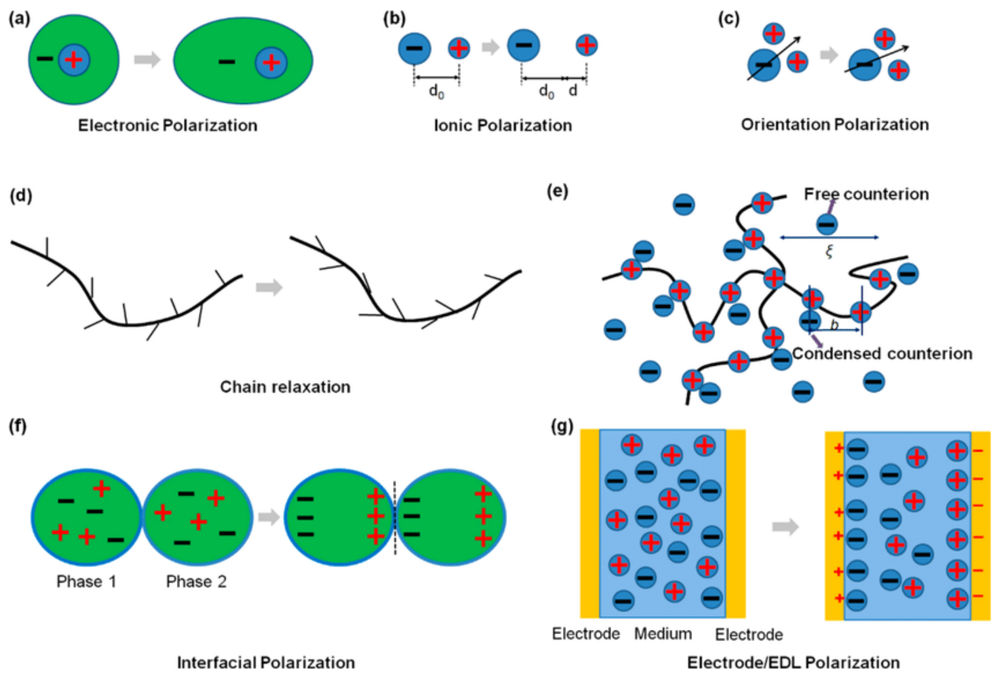
Importance of Polarization Process

1. Determines dielectric constant of materials.
2. Explains energy storage in capacitors.
3. Important in high-frequency and microwave applications.
4. Helps understand insulating behavior of solids.

The polarization process describes how a dielectric material responds to an applied electric field through the displacement and alignment of charges and dipoles. Different polarization mechanisms operate depending on material structure, temperature, and frequency. Understanding the polarization process is essential for explaining dielectric behavior and for designing materials used in electrical and electronic applications.

4.6 Electronic Polarization

Electronic polarization is the most fundamental type of polarization observed in dielectric materials. It occurs in **all atoms and molecules**, irrespective of whether the material is polar or non-polar. This type of polarization arises due to the displacement of the electron cloud relative to the nucleus when an external electric field is applied.



Concept of Electronic Polarization

In the absence of an electric field, the centre of positive charge (nucleus) and the centre of negative charge (electron cloud) coincide, and the atom behaves as an electrically neutral entity.

When an external electric field **E** is applied:

- The positively charged nucleus experiences a force in the direction of the field.
- The negatively charged electron cloud experiences a force opposite to the field.
- This causes a **slight separation** between the nucleus and the electron cloud.

As a result, an **induced electric dipole moment** is produced in the atom. This phenomenon is called **electronic polarization**.

Electronic Dipole Moment

The induced dipole moment **p** of an atom is given by:

$$p = \alpha_e E$$

where

α_e= electronic polarizability

E= applied electric field strength

Electronic polarizability depends on:

- Size of the atom
- Number of electrons
- Ease with which the electron cloud can be distorted

Larger atoms generally have higher electronic polarizability.

Electronic Polarization in Solids

In solids:

- Each atom develops an induced dipole moment.

- The net polarization **P** is the dipole moment per unit volume.

$$P = N\alpha_e E$$

where

N= number of atoms per unit volume

Characteristics of Electronic Polarization

1. Occurs in all dielectric materials.
2. Independent of temperature.
3. Very fast process.
4. Present even at optical frequencies.
5. Does not involve rotation or migration of atoms or ions.

Frequency Dependence

- Electronic polarization can follow very high frequency electric fields.
- It remains effective up to frequencies of the order of 10^{15} Hz.
- Beyond this range (ultraviolet region), polarization decreases sharply.

Restoring Force

The displacement of the electron cloud is opposed by:

- Coulomb attraction between nucleus and electrons.

Equilibrium is reached when:

- Force due to electric field equals restoring force.

Contribution to Dielectric Constant

Electronic polarization contributes significantly to:

- Dielectric constant at high frequencies.
- Optical properties such as refractive index.

For non-polar materials, electronic polarization is the **only** source of polarization.

Advantages of Electronic Polarization

- Instantaneous response to applied field.
- No energy loss due to thermal motion.
- Dominant polarization mechanism in high-frequency applications.

Electronic polarization arises from the distortion of the electron cloud relative to the nucleus under an applied electric field. It is universal, extremely rapid, and temperature independent. This mechanism plays a crucial role in determining the dielectric and optical properties of materials, especially at high frequencies.

4.7 Calculation of polarizability

Polarizability is a measure of how easily the charge distribution in an atom or molecule can be distorted by an external electric field. In electronic polarization, this distortion occurs due to the displacement of the electron cloud relative to the nucleus. The calculation of electronic polarizability can be carried out using a simple classical atomic model.

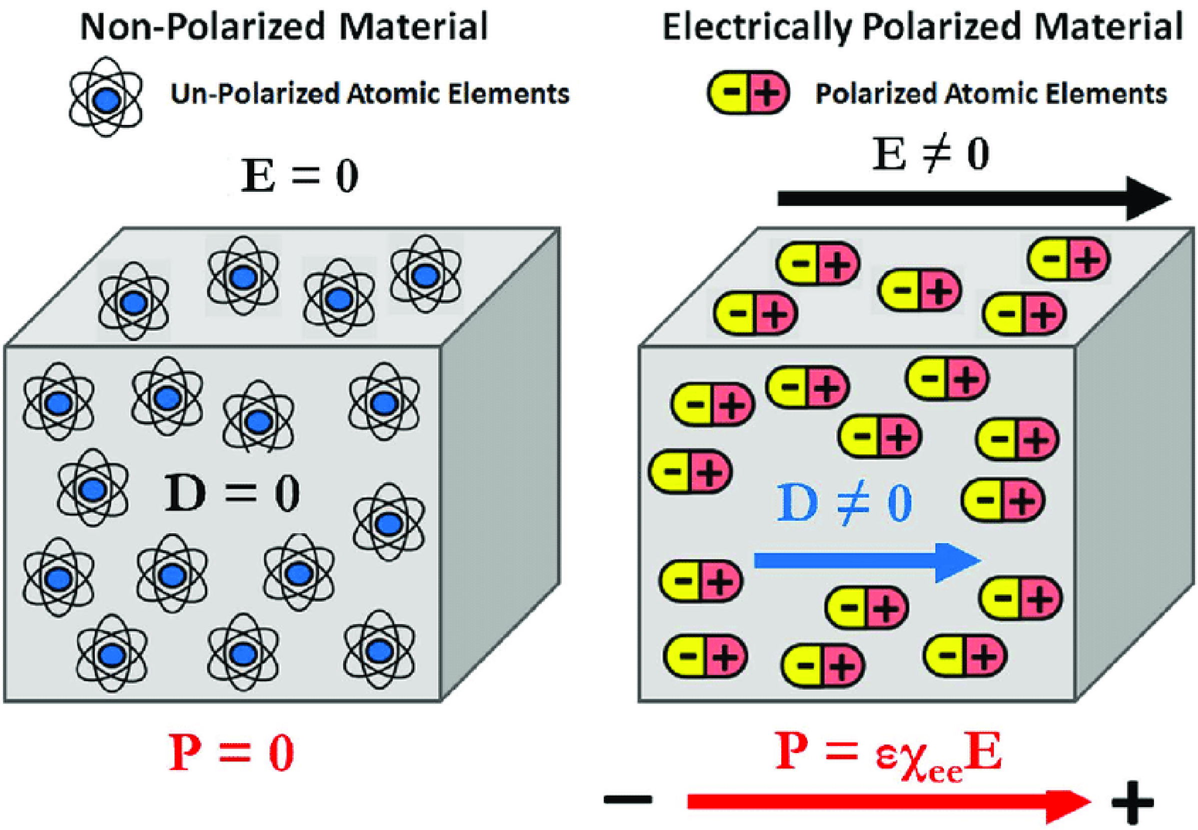


Figure 4. 3

Classical Model of an Atom

An atom is considered as:

- A positively charged nucleus located at the centre.
- A uniformly distributed spherical electron cloud of radius **r** surrounding the nucleus.

In the absence of an electric field, the centres of positive and negative charges coincide, and no dipole moment exists.

Effect of an External Electric Field

When an electric field **E** is applied:

- The nucleus experiences a force in the direction of the field.

- The electron cloud experiences a force opposite to the field.
- This causes a small displacement **x** between the nucleus and the electron cloud.

As a result, an induced electric dipole moment is produced.

Forces Acting on the Electron Cloud

1. Force due to External Electric Field

If **Ze** is the total electronic charge,

$$F_{\text{electric}} = ZeE$$

2. Restoring Force

The displacement of the electron cloud produces a restoring force due to electrostatic attraction between the nucleus and the displaced electron cloud.

For a uniformly charged sphere, the restoring force is:

$$F_{\text{restoring}} = \frac{Ze^2}{4\pi\varepsilon_0 r^3} x$$

Condition for Equilibrium

At equilibrium:

$$F_{\text{electric}} = F_{\text{restoring}}$$

$$ZeE = \frac{Ze^2}{4\pi\varepsilon_0 r^3} x$$

Solving for displacement **x**:

$$x = \frac{4\pi\varepsilon_0 r^3}{e} E$$

Induced Dipole Moment

The induced dipole moment **p** is given by:

$$p = Ze\,x$$

Substituting the value of **x**:

$$p = 4\pi\varepsilon_0 r^3 E$$

Electronic Polarizability

Electronic polarizability α_e is defined as the induced dipole moment per unit electric field:

$$\alpha_e = \frac{p}{E}$$

$$\alpha_e = 4\pi\varepsilon_0 r^3$$

Important Result

$$\boxed{\alpha_e = 4\pi\varepsilon_0 r^3}$$

This expression shows that:

- Electronic polarizability depends on the **volume of the atom**.
- Larger atoms have higher polarizability.
- Noble gases show increasing polarizability down the group.

Assumptions in the Calculation

1. Electron cloud is uniformly distributed.
2. Displacement is very small compared to atomic size.
3. Only electrostatic forces are considered.
4. Thermal effects are neglected.

Characteristics of Electronic Polarizability

- Independent of temperature.
- Present in all dielectric materials.
- Effective even at very high frequencies.
- Dominant polarization mechanism in non-polar solids.

The calculation of electronic polarizability using the classical atomic model shows that polarizability is directly proportional to the cube of the atomic radius. This result explains why larger atoms and ions are more easily polarized and highlights the fundamental role of electronic polarization in determining the dielectric and optical properties of materials.

4.8 Ionic, Orientational and space charge polarization

Apart from electronic polarization, dielectric materials exhibit other polarization mechanisms depending on their atomic or molecular structure. **Ionic polarization**, **orientational polarization**, and **space charge polarization** arise due to the displacement of ions, alignment of permanent dipoles, and accumulation of charges respectively. These mechanisms play an important role in determining the dielectric behavior of materials, especially at low and medium frequencies.

1. Ionic Polarization

Ionic polarization occurs in **ionic solids** when an external electric field causes relative displacement of positive and negative ions from their equilibrium positions.

Mechanism

- Ionic crystals consist of alternating positive and negative ions.

- In the absence of an electric field, the centres of positive and negative charges coincide.
- When an electric field is applied:
  - Positive ions shift slightly in the direction of the field.
  - Negative ions shift in the opposite direction.
- This relative displacement creates an induced dipole moment per unit cell.

Polarization Expression

The ionic polarization per unit volume is given by:

$$P_i = N\alpha_i E$$

where

N= number of ion pairs per unit volume

α_i= ionic polarizability

E= applied electric field

Characteristics

1. Occurs only in ionic crystals.
2. Depends on the mass and bonding strength of ions.
3. Slower than electronic polarization.
4. Effective up to infrared frequencies.
5. Independent of temperature.

Examples

NaCl, KCl, MgO

2. Orientational (Dipolar) Polarization

Definition

Orientational polarization occurs in **polar molecules** that possess a permanent electric dipole moment. It arises due to the partial alignment of these dipoles in the direction of an applied electric field.

Mechanism

- In the absence of an electric field, permanent dipoles are randomly oriented due to thermal motion.
- When an electric field is applied:
  - Dipoles tend to align along the field direction.
  - Thermal agitation opposes this alignment.
- Net polarization results from partial alignment of dipoles.

Polarization Expression

$$P_o = \frac{N\mu^2 E}{3kT}$$

where

N= number of dipoles per unit volume

μ= permanent dipole moment

k= Boltzmann constant

T= absolute temperature

Characteristics

1. Occurs only in polar dielectrics.
2. Strongly temperature dependent.

3. Decreases with increase in temperature.
4. Effective only at low frequencies.
5. Slower than electronic and ionic polarization.

Examples

H_2O, HCl, NH_3

3. Space Charge Polarization

Definition

Space charge polarization occurs due to the accumulation of charges at **interfaces, grain boundaries, defects, or electrode–dielectric boundaries** within a material.

Mechanism

- Mobile charge carriers (ions or electrons) migrate under an applied electric field.
- These charges get trapped at:
    - Grain boundaries
    - Phase boundaries
    - Impurities
    - Electrodes
- This accumulation produces large localized dipoles, leading to space charge polarization.

Characteristics

1. Occurs in heterogeneous dielectrics.
2. Dominant at very low frequencies.

3. Strongly dependent on material structure.
4. Very slow polarization process.
5. Causes high dielectric loss.

Examples

Ceramics, polymers, composite materials

Comparison of Polarization Mechanisms

Type of Polarization	Cause	Frequency Range	Temperature Dependence
Electronic	Electron cloud displacement	Very high	Independent
Ionic	Relative ion displacement	Infrared	Independent
Orientational	Dipole alignment	Low	Strong
Space charge	Charge accumulation	Very low	Strong

Ionic, orientational, and space charge polarization mechanisms arise due to different physical processes within dielectric materials. Ionic polarization is associated with ion displacement in crystals, orientational polarization with dipole alignment in polar molecules, and space charge polarization with charge accumulation at interfaces. Together with electronic polarization, these mechanisms determine the dielectric response of materials over a wide range of frequencies and temperatures.

4.9 Internal field

When a dielectric material is placed in an external electric field, it becomes polarized due to the displacement or alignment of charges. As a result of this polarization, an additional electric field is produced inside the dielectric. The **net electric field acting within the dielectric**, which differs from the applied field, is called the **internal field**.

Meaning of Internal Field

The internal field is the **effective electric field experienced by charges or dipoles inside a dielectric material**. It is not equal to the externally applied electric field because polarization of the dielectric produces an opposing field.

Origin of Internal Field

When a dielectric is polarized:

- Bound positive charges appear on one surface.
- Bound negative charges appear on the opposite surface.

These bound charges create an electric field inside the dielectric that **opposes the applied electric field**. This field is called the **polarization field**.

Relation between Applied Field and Internal Field

Let:

- E_0= applied electric field
- E_p= polarization field
- E_i= internal field

Then,

$$E_i = E_0 - E_p$$

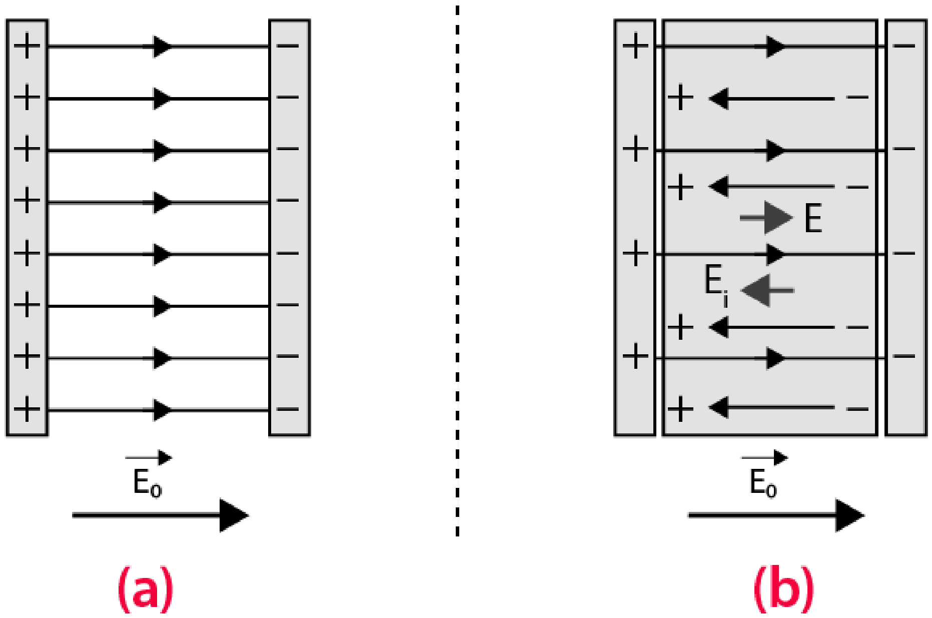


Figure 4. 4

Thus, the internal field is **less than the applied field**.

Polarization Field

The polarization field depends on the polarization $P$of the dielectric:

$$E_p = \frac{P}{\varepsilon_0}$$

Substituting,

$$E_i = E_0 - \frac{P}{\varepsilon_0}$$

This shows that as polarization increases, the internal field decreases.

Internal Field in a Dielectric Slab

For a dielectric slab placed between capacitor plates:

- The applied field tries to polarize the dielectric.
- The induced bound charges set up an opposing field.
- The net internal field determines the dielectric response.

The dielectric constant reduces the effective field inside the material.

Significance of Internal Field

1. Determines the degree of polarization in a dielectric.
2. Controls the dielectric constant of the material.
3. Influences electric susceptibility.
4. Important in understanding dielectric breakdown.
5. Affects optical and electrical properties of solids.

Difference between Internal Field and Local Field

- **Internal field**: Average field inside the dielectric.
- **Local field**: Field acting on an individual atom or molecule (includes contributions from nearby dipoles).

Internal field is a macroscopic quantity, while local field is microscopic.

Factors Affecting Internal Field

1. Nature of the dielectric material.
2. Polarization mechanism involved.
3. Strength of applied electric field.
4. Geometry and shape of the dielectric.

5. Dielectric constant.

The internal field is the effective electric field within a dielectric material that results from the combined action of the applied electric field and the opposing polarization field. Since polarization reduces the net field inside the dielectric, the internal field plays a crucial role in determining the electrical behavior and dielectric properties of materials.

4.10 Clausius-Mosotti relation

The Clausius–Mossotti relation establishes an important connection between the **macroscopic dielectric constant** of a material and the **microscopic polarizability of its atoms or molecules**. It provides a bridge between atomic-level behavior and bulk dielectric

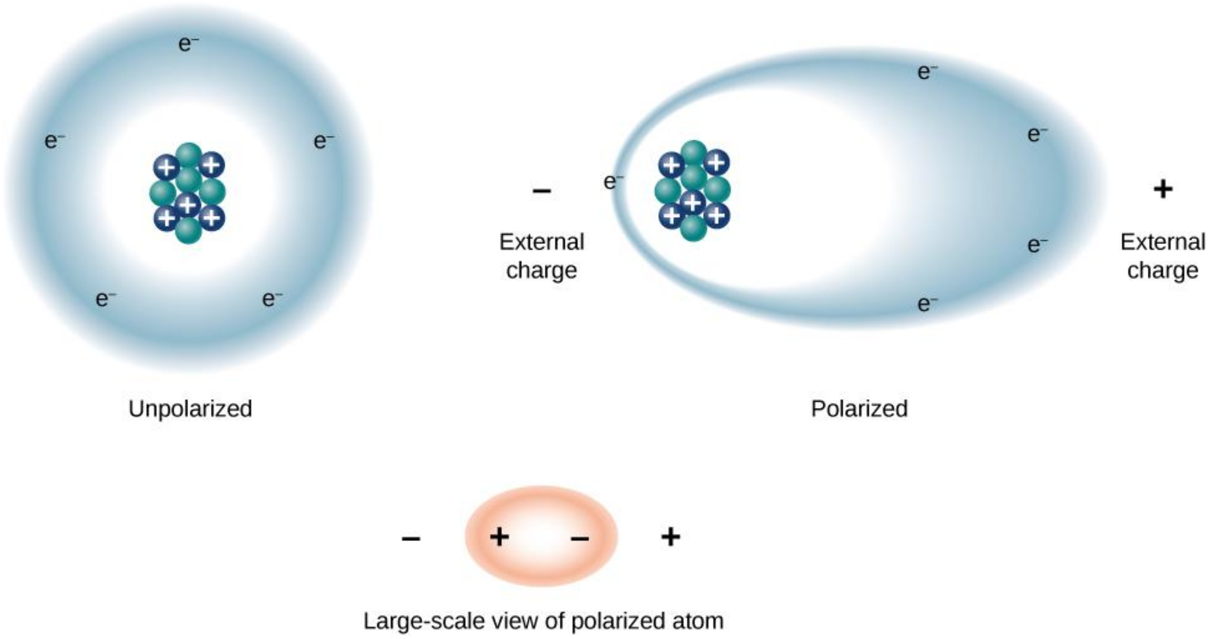


Figure 4. 5

properties and is applicable mainly to **isotropic, non-polar dielectric materials**.

Need for Clausius–Mossotti Relation

- Dielectric constant (ε_r) is a macroscopic property.
- Polarizability (α) is a microscopic property.

- To relate these two quantities, the concept of **local electric field** acting on an atom is introduced.
- Clausius–Mossotti relation arises by considering the **Lorentz local field** inside a dielectric.

Local Electric Field (Lorentz Field)

The local electric field E_L acting on an atom inside a dielectric is given by:

$$E_L = E_i + \frac{P}{3\varepsilon_0}$$

where

E_i= internal electric field

P= polarization

ε_0= permittivity of free space

Polarization and Polarizability

The induced dipole moment p of an atom is:

$$p = \alpha E_L$$

Polarization P is dipole moment per unit volume:

$$P = Np = N\alpha E_L$$

where

N= number of atoms per unit volume

Substituting the value of E_L:

$$P = N\alpha \left(E_i + \frac{P}{3\varepsilon_0}\right)$$

Derivation of Clausius–Mossotti Relation

Rearranging,

$$P - \frac{N\alpha}{3\varepsilon_0} P = N\alpha E_i$$

$$P\left(1 - \frac{N\alpha}{3\varepsilon_0}\right) = N\alpha E_i$$

$$P = \frac{N\alpha E_i}{1 - \frac{N\alpha}{3\varepsilon_0}}$$

But polarization is also related to dielectric constant as:

$$P = \varepsilon_0(\varepsilon_r - 1)E_i$$

Equating the two expressions for $P$and simplifying, we get:

$$\boxed{\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}}$$

This is the **Clausius–Mossotti relation**.

Significance of the Relation

1. Connects microscopic polarizability with macroscopic dielectric constant.
2. Useful in estimating atomic or molecular polarizability.
3. Explains dielectric behavior of non-polar solids and gases.
4. Important in optical and solid-state physics.
5. Helps understand refractive index of materials.

Validity of Clausius–Mossotti Relation

The relation is valid when:

- The dielectric is isotropic.
- Only electronic polarization is significant.
- Interactions between neighboring dipoles are negligible.
- The material is non-polar.

It is not accurate for strongly polar or highly dense materials.

Applications

- Determination of polarizability of atoms and molecules.
- Study of dielectric and optical properties of solids.
- Estimation of dielectric constant at high frequencies.
- Analysis of insulating materials.

Limitations

1. Not applicable to polar dielectrics with strong dipole interactions.
2. Assumes uniform local field.
3. Neglects short-range atomic interactions.
4. Less accurate for solids with complex structures

The Clausius–Mossotti relation provides a fundamental link between atomic polarizability and the dielectric constant of a material. By incorporating the concept of local electric field, it successfully explains how microscopic charge displacement contributes to macroscopic dielectric behavior, making it a cornerstone relation in dielectric and solid-state physics.

4.11 Frequency dependence of dielectric constant

The dielectric constant of a material is not a fixed quantity; it varies with the frequency of the applied alternating electric field. This variation arises because different polarization mechanisms respond differently to changes in frequency. The study of how dielectric constant changes with frequency is known as **dielectric dispersion**.

Dielectric Constant and Polarization

The dielectric constant ε_r of a material is related to polarization as:

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E}$$

Since polarization P depends on the ability of dipoles and charges to follow the applied electric field, the dielectric constant becomes frequency dependent.

Polarization Mechanisms and Frequency

Different polarization mechanisms operate over different frequency ranges:

1. **Space Charge Polarization**
2. **Orientational (Dipolar) Polarization**
3. **Ionic Polarization**
4. **Electronic Polarization**

Each mechanism has a characteristic response time. As frequency increases, slower mechanisms fail to respond, leading to a decrease in the dielectric constant.

Variation of Dielectric Constant with Frequency

Low Frequency Region

- All polarization mechanisms contribute.
- Space charge, orientational, ionic, and electronic polarizations are active.
- Dielectric constant has its **maximum value**.

Intermediate Frequency Region

- Space charge polarization disappears first.
- Orientational polarization fails at higher frequencies due to molecular inertia.

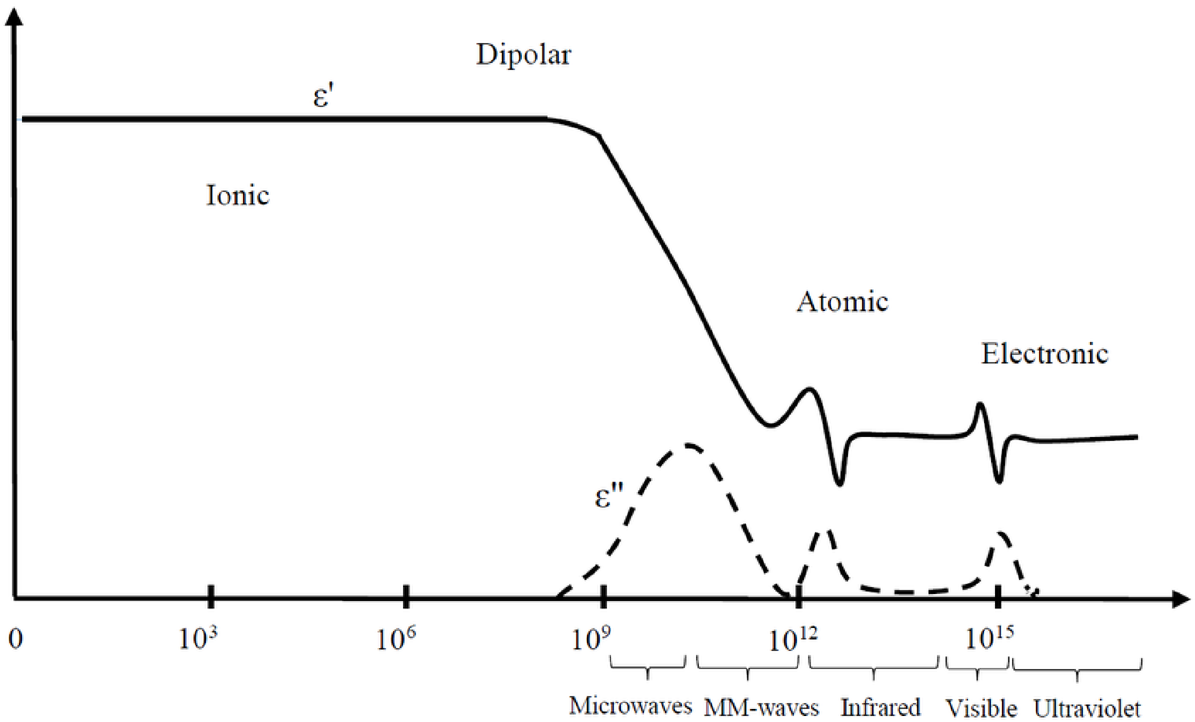


Figure 4. 6

- Dielectric constant decreases in steps.

High Frequency Region

- Ionic polarization ceases to respond.
- Only electronic polarization remains active.

- Dielectric constant becomes nearly constant and low.

Very High Frequency (Optical Region)

- Even electronic polarization fails beyond optical frequencies.
- Dielectric constant approaches unity.

Dielectric Dispersion Curve

The typical dielectric dispersion curve shows:

- Stepwise decrease in dielectric constant with increasing frequency.
- Each step corresponds to the loss of a polarization mechanism.
- Flat regions represent frequency ranges where polarization remains constant.

Dielectric Relaxation

- The lag between polarization and applied electric field causes dielectric relaxation.
- It is prominent in orientational polarization.
- Associated with dielectric loss and energy dissipation.

Importance of Frequency Dependence

1. Determines suitability of dielectrics for high-frequency applications.
2. Important in microwave and optical devices.
3. Helps in understanding insulation behavior.
4. Essential in capacitor and communication system design.

The dielectric constant decreases with increasing frequency because different polarization mechanisms fail to respond beyond their characteristic frequency ranges. At low frequencies, all polarization mechanisms contribute, resulting in a high dielectric constant, whereas at very high frequencies, only electronic polarization remains. Understanding the frequency dependence of dielectric constant is crucial for the effective application of dielectric materials in electrical and electronic systems.

4.12 Dielectric loss

When a dielectric material is subjected to an alternating electric field, polarization does not occur instantaneously. Due to this time lag between the applied electric field and the resulting polarization, part of the electrical energy supplied to the dielectric is dissipated as heat. This loss of energy is known as **dielectric loss**. It is an important factor in the performance of dielectric materials, especially at high frequencies.

Cause of Dielectric Loss

Dielectric loss occurs due to:

- Lag of polarization behind the alternating electric field.
- Internal friction during dipole rotation.
- Migration of charges in space charge polarization.
- Imperfect response of polarization mechanisms.

The loss is mainly associated with **orientational and space charge polarization**, which are slow processes.

Complex Permittivity

The dielectric behavior of a material under an AC field is represented by complex permittivity:

$$\varepsilon^* = \varepsilon' - j\varepsilon''$$

where

ε'= dielectric constant (energy storage component)

ε''= dielectric loss component (energy dissipation component)

Loss Tangent (Dielectric Loss Factor)

Dielectric loss is commonly expressed using the **loss tangent** or **dissipation factor**:

$$\tan\delta = \frac{\varepsilon''}{\varepsilon'}$$

where

δ= loss angle

- Smaller $\tan\delta$→ better dielectric (low energy loss)
- Larger $\tan\delta$→ poor dielectric (high energy loss)

Power Loss in a Dielectric

The power loss per unit volume in a dielectric subjected to an AC electric field is given by:

$$P = \omega\varepsilon_0\varepsilon' E^2 \tan\delta$$

where

ω= angular frequency

E= electric field strength

Frequency Dependence of Dielectric Loss

- At low frequencies, dielectric loss is small.
- Near the relaxation frequency of orientational polarization, dielectric loss becomes maximum.
- At very high frequencies, dielectric loss decreases as polarization mechanisms fail to respond.

This variation is known as **dielectric relaxation loss**.

Temperature Dependence

- Dielectric loss increases with temperature in polar dielectrics.
- Increased thermal agitation enhances dipole motion, increasing energy dissipation.
- In non-polar dielectrics, temperature dependence is weak.

Sources of Dielectric Loss

1. Dipole rotation losses.
2. Ionic displacement losses.
3. Space charge migration losses.
4. Leakage current losses.

Effects of Dielectric Loss

1. Heating of dielectric materials.
2. Reduction in efficiency of capacitors.
3. Limits use of materials in high-frequency applications.
4. Causes dielectric breakdown in extreme cases.

Low-Loss Dielectric Materials

Materials with low dielectric loss are preferred for high-frequency applications:

- Mica
- Quartz
- Teflon
- Polystyrene

Applications

- Selection of insulating materials.
- Design of capacitors and RF components.
- Microwave and communication systems.
- High-voltage insulation engineering.

Dielectric loss represents the energy dissipated as heat in a dielectric material when subjected to an alternating electric field. It arises due to the delayed response of polarization mechanisms and is characterized by the loss tangent. Understanding dielectric loss is essential for choosing suitable dielectric materials, particularly in high-frequency and high-power electrical applications.

4.13 Effect Of Temperature On Dielectric Constant

The dielectric constant of a material depends not only on frequency and electric field but also significantly on **temperature**. Temperature affects the ability of different polarization mechanisms to operate within a dielectric. Since each polarization mechanism responds differently to thermal energy, the dielectric constant may increase or decrease with temperature depending on the nature of the material.

Dielectric Constant and Polarization

The dielectric constant $\varepsilon_r$is related to polarization $P$by:

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E}$$

Any factor that changes polarization will therefore affect the dielectric constant. Temperature mainly influences **orientational** and **space charge polarization**.

Effect of Temperature on Different Polarization Mechanisms

1. Electronic Polarization

- Caused by displacement of the electron cloud.
- Atomic dimensions are almost unaffected by temperature.
- Hence, electronic polarization is **independent of temperature**.
- Dielectric constant contribution remains nearly constant.

2. Ionic Polarization

- Occurs due to relative displacement of ions in ionic solids.
- Slightly affected by lattice expansion at high temperatures.
- Dielectric constant shows **very weak temperature dependence**.
- Effect is usually negligible in most practical cases.

3. Orientational (Dipolar) Polarization

- Strongly affected by temperature.
- As temperature increases:
  - Thermal agitation increases.
  - Alignment of permanent dipoles becomes difficult.

- Orientational polarization decreases.

- Hence, dielectric constant **decreases with increase in temperature**.

Mathematically,

$$P_o \propto \frac{1}{T}$$

4. Space Charge Polarization

- Increases with temperature initially due to enhanced mobility of charge carriers.
- At higher temperatures, random motion dominates and polarization may decrease.
- Shows complex temperature dependence.

Temperature Dependence in Polar and Non-Polar Dielectrics

Non-Polar Dielectrics

- Only electronic polarization is present.
- Dielectric constant is nearly **independent of temperature**.
- Example: Noble gases, polyethylene.

Polar Dielectrics

- Both electronic and orientational polarizations are present.
- Dielectric constant **decreases with increase in temperature**.
- Example: Water, alcohols, ammonia.

Curie Law for Polar Dielectrics

For many polar dielectrics, the temperature dependence of dielectric constant follows:

$$\varepsilon_r = A + \frac{B}{T}$$

where

A and B are constants.

- At low temperatures, dipole alignment is easier → higher dielectric constant.
- At high temperatures, thermal agitation dominates → lower dielectric constant.
- In some materials, dielectric constant shows a peak near phase transitions.

Importance of Temperature Effect

1. Selection of dielectric materials for temperature-sensitive applications.
2. Design of capacitors and insulating systems.
3. Understanding dielectric stability.
4. High-temperature electrical insulation engineering.

The dielectric constant of a material varies with temperature due to changes in polarization mechanisms. While electronic and ionic polarizations are almost temperature independent, orientational and space charge polarizations are strongly influenced by temperature. As a result, polar dielectrics generally show a decrease in dielectric constant with increasing temperature, whereas non-polar dielectrics remain nearly unaffected. Understanding this behavior is essential for reliable use of dielectrics in practical applications.

Unit 5: Ferroelectric & Superconducting Properties of Materials

1. Introduction
2. Ferroelectric effect
3. Weiss Law
4. Ferroelectric domains
5. Elementary band theory
6. Band gap (No derivation)
7. Hall effect
8. Measurement of conductivity (four probe method)
9. Hall coefficient
10. Superconductivity
11. General properties of superconducting materials
12. Critical temperature
13. Critical magnetic field
14. Meissner effect
15. Isotope effect
16. Type-I & Type-II Superconductors
17. London's equation and penetration depth

5.1 Introduction

Ferroelectric Materials

Ferroelectricity is basically the electric version of ferromagnetism. These materials have a built-in electric dipole moment (polarization) even when no external electric field is applied.

You can *reverse* this polarization by applying an external electric field — like flipping a tiny electric switch inside the material.

This happens because the atoms inside the crystal don't sit perfectly symmetrical. A slight "off-center" shift in ions creates a permanent dipole. When you apply an electric field, these dipoles align or reverse direction, giving rise to a **hysteresis loop**, which is the signature property of ferroelectrics.

Common examples: $BaTiO_3$ (barium titanate), Rochelle salt, PZT ceramics.
Applications: Non-volatile memories (FeRAM), sensors, actuators, piezoelectric devices, capacitors, and transducers.

Ferroelectric properties typically disappear above a specific temperature called the **Curie temperature**, where the material becomes paraelectric (no permanent polarization).

Superconducting Materials

Superconductivity is one of the most mind-blowing states of matter. When certain materials are cooled below a critical temperature (Tc), their electrical resistance suddenly drops to **zero**. Like literally **zero** — meaning current can flow forever without losing energy.

Another flagship property is the **Meissner effect**, where the material kicks out all magnetic fields from its interior and becomes perfectly diamagnetic. This is why superconductors can levitate magnets — that viral YouTube-style floating experiment.

Superconductivity arises from electrons pairing up into **Cooper pairs**, which move through the crystal without scattering. This highly ordered quantum state gives the material its magical properties.

Types:

- **Type I superconductors:** Pure metals (e.g., Pb, Hg) – complete Meissner effect.
- **Type II superconductors:** Alloys/ceramics (e.g., NbTi, YBCO) – allow magnetic flux tubes (vortices) to penetrate.

Applications: MRI machines, maglev trains, SQUIDs (insanely sensitive magnetic sensors), particle accelerators, and lossless power transmission.

Ferroelectricity and superconductivity both come from **collective behavior** of electrons inside solids. They show how small atomic shifts or electron pairings can create large-scale, technological game-changing effects. Plus, modern research even explores multiferroics and coexistence of ferroic properties and superconductivity.

5.2 Ferroelectric effect

Introduction

Ferroelectricity refers to the property of certain dielectric materials that exhibit a **spontaneous electric polarization** even without the application of an external electric field. This spontaneous polarization is not permanent; it can be **reversed** when an electric field of sufficient magnitude is applied. Because of this switchable polarization, ferroelectricity is considered the electrical analogue of ferromagnetism. Materials exhibiting this behavior are known as **ferroelectric materials**.

Origin of Ferroelectricity

Ferroelectric behavior originates from a **non-centrosymmetric crystal structure**, in which the centers of positive and negative charges do not coincide. This asymmetry arises due to a structural phase transition that occurs at a characteristic temperature known as the **Curie temperature (Tc)**.

Above the Curie temperature, the material remains in a symmetric **paraelectric** phase with no permanent dipole moment. When cooled below Tc, the unit cell becomes distorted. This distortion shifts certain ions from their equilibrium positions, creating an internal dipole moment and giving rise to **spontaneous polarization**. A well-known example is **barium titanate ($BaTiO_3$)**, where a slight displacement of the Ti^{4+} ion within the oxygen octahedron breaks the symmetry and produces a permanent dipole.

Domains and Domain Alignment

Ferroelectric materials consist of small regions called **domains**, in which electric dipoles are oriented in the same direction. In an unpolarized state, domains are randomly arranged, resulting in zero net polarization. When an external electric field is applied, domains begin to align with the field direction. Domains already aligned with the field grow in size, while those opposing it shrink. In some cases, new domains may form that align with the applied field.

Even after the external field is removed, a portion of the polarization remains. This is known as **remanent polarization**, a key characteristic of ferroelectric materials.

Polarization–Electric Field (P–E) Hysteresis

The most important signature of ferroelectric materials is the **P–E hysteresis loop**. When the electric field is cycled, the polarization does not follow the same path during increasing and decreasing fields; instead, it forms a closed loop.

This loop shows several important parameters:

- **Saturation polarization (Ps):** The maximum polarization when all dipoles are fully aligned.
- **Remanent polarization (Pr):** The polarization that remains after the external field is reduced to zero.
- **Coercive field (Ec):** The reverse electric field needed to bring the net polarization back to zero.

This hysteresis behavior demonstrates that the polarization switching in ferroelectrics involves energy barriers and domain reorientation.

Curie Temperature and Phase Transition

Every ferroelectric material has a characteristic **Curie temperature**, above which ferroelectric properties disappear. At this temperature, the material undergoes a structural phase transition from the ferroelectric phase (with spontaneous polarization) to the paraelectric phase (with no spontaneous polarization).

This transition is often accompanied by changes in crystal symmetry and dielectric constant. Near the Curie point, the dielectric constant typically becomes very high, making ferroelectrics useful in capacitor applications.

Applications of Ferroelectric Materials

Ferroelectric materials are widely used in modern technology due to their unique property of switchable polarization. They are essential in:

- **Non-volatile FeRAM memory**
- **Capacitors with high dielectric constants**

- **Piezoelectric and pyroelectric sensors**
- **Actuators and transducers**
- **Electro-optic devices**

Their ability to retain polarization states without power makes them particularly important for low-power memory technologies.

5.3 Weiss Law

In ferromagnetic materials, atomic magnetic moments tend to align spontaneously due to strong internal interactions. This alignment produces spontaneous magnetization even without an external magnetic field. Pierre Weiss introduced the concept of a **molecular field**, an internal field that acts on each magnetic moment and encourages alignment. Using this idea, Weiss formulated a temperature-dependent law for magnetic susceptibility in the paramagnetic region above the Curie temperature. This relationship is known as the **Curie–Weiss law**.

Weiss Molecular Field Concept

Weiss proposed that in addition to the external magnetic field H, each atom experiences an internal field proportional to the magnetization M:

$$H_{\text{eff}} = H + \lambda M$$

Here, $\lambda$is the Weiss constant. This internal field represents the interaction among neighboring magnetic moments.

Magnetization in the Paramagnetic Region

For a paramagnetic system, the magnetization produced by an effective field follows the Curie law:

$$M = \frac{C}{T} H_{\text{eff}}$$

Substituting Weiss's effective field:

$$M = \frac{C}{T}(H + \lambda M)$$

This becomes:

$$M = \frac{C}{T} H + \frac{C\lambda}{T} M$$

Bring all M-terms to one side:

$$M - \frac{C\lambda}{T} M = \frac{C}{T} H$$

Factor out M:

$$M\left(1 - \frac{C\lambda}{T}\right) = \frac{C}{T} H$$

Expression for Susceptibility

Magnetic susceptibility is defined as:

$$\chi = \frac{M}{H}$$

Divide both sides of the magnetization equation by H:

$$\frac{M}{H} = \frac{C/T}{1 - C\lambda/T}$$

$$\chi = \frac{C}{T - C\lambda}$$

Define the Curie temperature:

$$T_C = C\lambda$$

Substitute:

$$\chi = \frac{C}{T - T_C}$$

This is the **Curie–Weiss law**, valid for **T > T** $_C$.

Final Result

$$\boxed{\chi = \frac{C}{T - T_C}}$$

This equation shows that magnetic susceptibility increases strongly as the temperature approaches the Curie temperature from above. At $T = T_C$, the susceptibility tends toward infinity, indicating the onset of spontaneous magnetization.

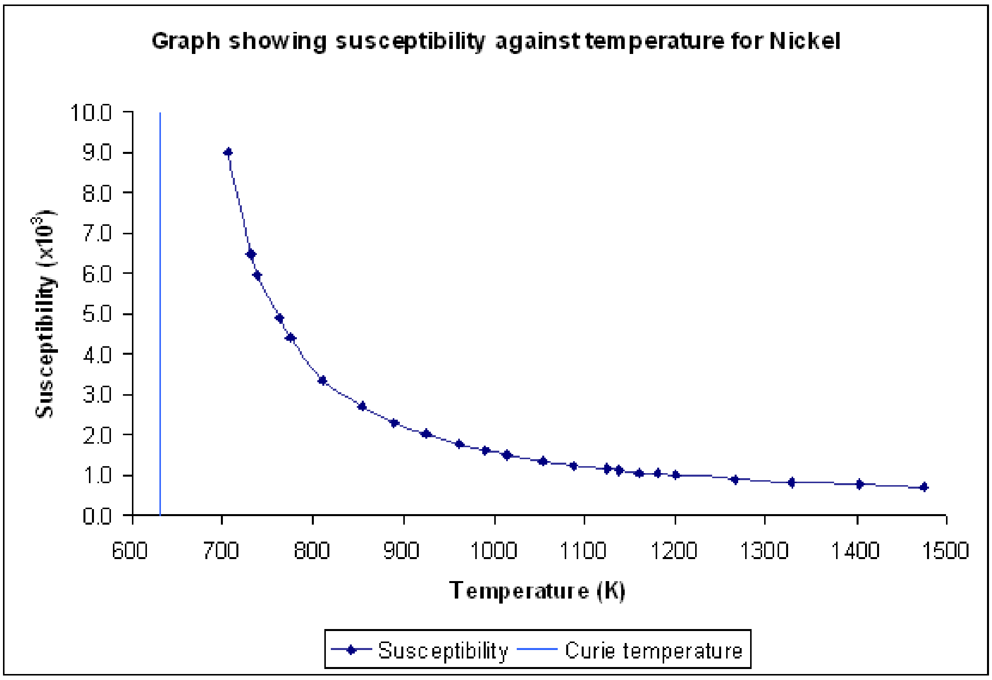


Figure 5. 1

5.4 Ferroelectric domains

Ferroelectric materials possess a spontaneous electric polarization that can exist even without an external electric field. This polarization does not occur uniformly throughout the entire crystal. Instead, the crystal splits into small regions within which all electric dipoles are aligned in the same direction. These regions are known as **ferroelectric domains**. The arrangement, interaction, and motion of these domains are central to understanding ferroelectric behavior, polarization switching, hysteresis, and device applications.

Formation of Domains

A single large region with uniform polarization would create extremely strong **depolarizing fields**, which are energetically unfavorable. To minimize the internal electrostatic energy, the crystal naturally divides into multiple domains with different polarization orientations.

Inside each domain, the dipoles are nearly perfectly aligned.

Between domains, boundaries form where the direction of polarization changes. These boundaries are known as **domain walls**, commonly designated as:

- 180° domain walls: the polarization reverses direction
- 90° domain walls: the polarization rotates by 90° due to structural distortion

These walls allow the crystal to lower its energy by arranging polarization directions in a pattern that reduces the overall electrostatic field.

Domain Structure

The structure of ferroelectric domains depends on the crystal symmetry and internal stresses. For example, in $BaTiO_3$, the tetragonal phase commonly contains both 180° and 90° domains arranged in a striped or lamellar pattern. The boundaries between these regions are atomically thin but mechanically stable.

The domain configuration is not random. It reflects the balance among several energies: elastic energy, electrostatic energy, and domain-wall energy. The competition among these determines the shape, size, and orientation of domains.

Domain Wall Motion

When an external electric field is applied to a ferroelectric material, domain walls begin to move. Domains aligned with the field grow, while those oppositely aligned shrink. This movement of domain walls is responsible for:

- polarization switching
- hysteresis behavior
- remanent polarization

The ease with which walls move determines important material properties such as coercive field, dielectric constant, and fatigue endurance. In many ferroelectric devices, controlling domain-wall motion is essential for stable performance.

Role of Domains in Polarization Switching

Polarization reversal in a ferroelectric material does not occur by flipping individual dipoles independently. Instead, the reversal happens through a **collective process**:

- new domains nucleate with polarization aligned to the applied field
- these domains expand by moving domain walls
- eventually, they replace domains with the opposite orientation

This mechanism explains the characteristic **P–E hysteresis loop**, where energy is expended in moving domain walls and overcoming pinning centers such as defects, impurities, or grain boundaries.

Applications Related to Ferroelectric Domains

Ferroelectric domains play a crucial role in many modern technologies. Devices that rely on domain behavior include:

- ferroelectric RAM (FeRAM)
- non-volatile memories
- actuators and transducers
- piezoelectric sensors
- tunable capacitors
- electro-optic modulators

Because domain dynamics control switching speed, stability, and endurance, the study of domain structures is essential in designing high-performance ferroelectric devices.

5.5 Elementary band theory

Introduction

Elementary band theory explains how the arrangement of electrons in solids determines whether a material behaves as a conductor, semiconductor, or insulator. When atoms come together to form a solid, their discrete energy levels interact and split into closely spaced levels, forming continuous ranges of energy called **energy bands**. The behavior of electrons in these bands governs the electrical and optical properties of the material.

Formation of Bands

In an isolated atom, electrons occupy well-defined, discrete energy levels. However, when a huge number of atoms combine to form a crystal, the outer electron wavefunctions overlap. Because two electrons cannot occupy the same energy state in a system, each atomic energy level splits into a very large number of allowed states. These extremely dense levels merge into broad regions known as **energy bands**.

The most important bands are:

- the **valence band**, containing the highest-energy electrons bound to atoms
- the **conduction band**, which contains free electrons able to move throughout the solid

Between these bands is a region where no electron states exist, known as the **band gap**.

Band Gap and Material Classification

The electrical nature of a solid depends on the size of the band gap and the occupancy of the bands.

In **conductors**, the valence band overlaps with the conduction band or is partially filled. Because electrons require no additional energy to move into available states, conduction occurs readily.

In **semiconductors**, a small band gap exists (typically < 3 eV). Electrons can be thermally excited from the valence band to the conduction band. This allows controlled conduction, sensitive to temperature and doping.

In **insulators**, the band gap is large (usually > 5 eV). Electrons cannot easily jump to the conduction band, preventing current flow under normal conditions.

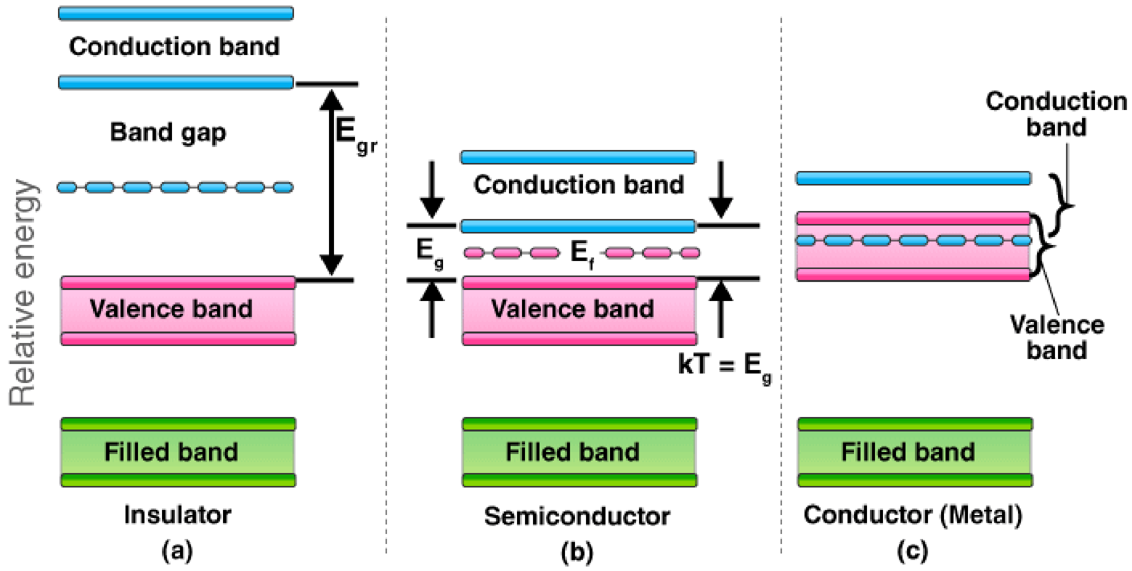


Figure 5. 2

Fermi Energy and Occupation of Bands

The **Fermi energy** is the energy level at which the probability of electron occupancy is 50% at absolute zero. It represents the highest filled level at 0 K.

In conductors, the Fermi level lies within a band, allowing free movement of electrons.

In semiconductors and insulators, the Fermi level lies inside the band gap, so electrons must be excited across the gap for conduction to occur.

The position of the Fermi level plays a crucial role in determining a material's conductivity, optical absorption, and carrier concentration.

Effective Mass of Electrons

Electrons in a crystal do not behave like free electrons. Instead, due to interactions with the periodic lattice potential, they behave as if they have a different inertial mass, known as the **effective mass**.

A positive effective mass corresponds to electrons, while a negative effective mass of the valence band top is interpreted as **holes**. These holes act as positive charge carriers, especially important in semiconductors.

Band Theory and Real Materials

Elementary band theory provides a simplified but powerful picture of how electronic states are organized in solids. It successfully explains:

- why metals conduct electricity
- how semiconductors respond to doping
- why insulators do not conduct
- optical absorption edges
- thermoelectric and photonic properties

More advanced theories add corrections using quantum mechanics and take into account electron–electron interactions, but elementary band theory remains the foundation.

5.6 Band gap

Band Gap – Explanation (Textbook Style)

The **band gap** is the energy difference between the **valence band** (the highest energy band filled with electrons under normal conditions) and the **conduction band** (the next higher band where electrons are free to move and conduct electricity). No allowed energy states exist inside this gap, meaning an electron **cannot remain** in the band-gap region.

Physical Meaning

For an electron to move from the valence band to the conduction band, it must gain an energy equal to or greater than the band gap. Once in the conduction band, the electron becomes mobile and contributes to electrical conduction.
A larger band gap means the electrons need more external energy to cross over; a smaller band gap makes the transition easier.

Band Gap and Types of Materials

In **conductors**, the valence and conduction bands overlap. Because there is no band gap, electrons move freely, giving high electrical conductivity.

In **semiconductors**, a small band gap exists. Only moderate energy input (like heat or light) is needed to excite electrons. Silicon and germanium are classic examples, where electrons can cross the gap at room temperature.

In **insulators**, the band gap is very large. Normal thermal energy is not enough for electrons to reach the conduction band, so conductivity is nearly zero.

Role of Band Gap in Charge Carriers

When an electron jumps to the conduction band, it leaves behind a **hole** in the valence band. Both electrons in the conduction band and holes in the valence band act as charge carriers. The band-gap size directly controls how many such carriers are available at a given temperature.

Summary

The band gap is the key energy barrier that determines whether a solid behaves as a conductor, semiconductor, or insulator. A zero band gap leads to metallic conduction, a small band gap results in semiconductor behavior, and a large band gap produces insulating properties.

5.7 Hall effect

Consider a rectangular conductor carrying an electric current along the x-direction. When a magnetic field is applied along the z-direction, the moving charge carriers inside the conductor experience a sideways magnetic force. This force pushes the carriers either upward or downward depending on the sign of their charge.

Lorentz Force

A charge $q$moving with drift velocity $v_d$in a magnetic field **B**experiences a magnetic force

$$F_B = qv_dB$$

This force acts perpendicular to both the current and the magnetic field.

Formation of Hall Electric Field

As the carriers accumulate on one side, an electric field E_H is produced across the width of the conductor. This electric field produces an opposing electric force

$$F_E = qE_H$$

Equilibrium is reached when the magnetic and electric forces balance:

$$qE_H = qv_dB$$

$$E_H = v_dB$$

Relation Between Drift Velocity and Current Density

The drift velocity of charge carriers is linked to the current density J:

$$J = nqv_d$$

where

n= number of charge carriers per unit volume

q= charge of each carrier

v_d= drift velocity

Solving for drift velocity:

$$v_d = \frac{J}{nq}$$

Hall Electric Field in Terms of Current Density

Substituting this into the earlier relation:

$$E_H = \frac{J}{nq}B$$

Definition of Hall Coefficient

The Hall coefficient $R_H$is defined as:

$$R_H = \frac{E_H}{JB}$$

Substitute $E_H = \frac{J}{nq}B$:

$$R_H = \frac{1}{nq}$$

This is the fundamental expression for the Hall coefficient.

Hall Voltage

If the width of the conductor is w, the Hall voltage is:

$$V_H = E_H\, w$$

$$V_H = \left(\frac{J}{nq}B\right)w$$

Because $J = I/(tw)$, where $t$is thickness:

$$V_H = \frac{IB}{nqt}$$

This is the standard formula used in experiments to measure carrier concentration in semiconductors.

Summary of the Derivation

The Hall effect arises from the balance of magnetic and electric forces acting on charge carriers. This balance leads directly to the Hall electric field, the Hall voltage, and the Hall coefficient. The final expression

$$R_H = \frac{1}{nq}$$

shows that the Hall effect is an effective method to determine both the **sign** and **concentration** of charge carriers in a material.

5.8 Measurement of conductivity (four probe method)

Four-Probe Method

Measuring electrical conductivity requires determining how easily charges move inside a material. A direct two-probe method often fails because the measured resistance includes probe-to-sample contact resistance and wire resistance, which can be much larger than the intrinsic resistance of the sample. This is especially problematic in low-resistivity materials such as semiconductors and metals.

The four-probe method solves this by using four equally spaced, collinear probes. A constant current is supplied through the **two outer probes**, while the **voltage is measured across the two inner probes**. Because the voltmeter has extremely high input impedance, almost no current flows through the inner probes. As a result, the voltage drop measured is purely the potential difference across the sample material, unaffected by contact resistances.

Current Distribution and Potential inside the Sample

When current enters the sample from the outer probe, it spreads out radially. For a semi-infinite sample, the current flows hemispherically from the probe tip. The current density at

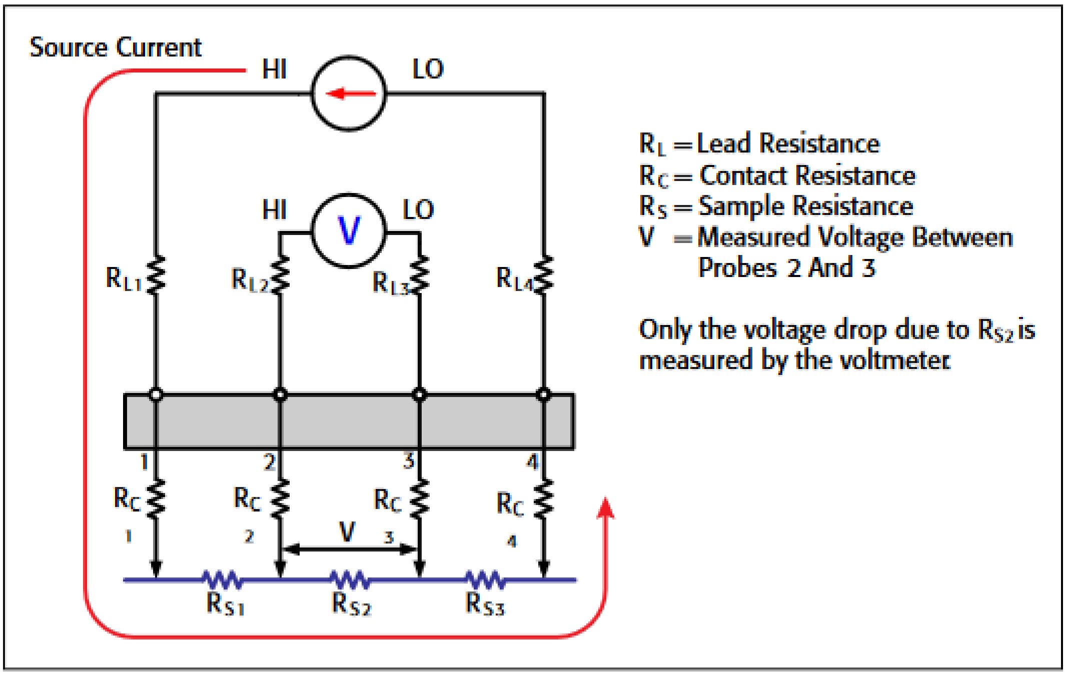


Figure 5. 3

any distance r from the probe is:

$$J = \frac{I}{2\pi r^2}$$

The associated electric field at the same point is:

$$E = \rho J = \frac{\rho I}{2\pi r^2}$$

where $\rho$is the resistivity of the sample.

The electric field is related to potential by:

$$E = -\frac{dV}{dr}$$

Thus, the potential difference between two distances r_1 and r_2 is:

$$\Delta V = \int_{r_1}^{r_2} E\, dr = \frac{\rho I}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Applying to Four Equally Spaced Probes

Let the spacing between the probes be s.

For the two inner probes located at distances s and 2s from the current-injecting probe, the voltage difference becomes:

$$\Delta V = \frac{\rho I}{2\pi s}$$

This is the fundamental working equation of the four-probe method for a semi-infinite thick sample.

Rearranging the above equation, we get the resistivity:

$$\rho = 2\pi s \frac{\Delta V}{I}$$

Conductivity from Resistivity

Electrical conductivity σ is defined as:

$$\sigma = \frac{1}{\rho}$$

Thus, once resistivity is found using the measured ΔV and applied current I, the conductivity follows directly.

Why the Four-Probe Method is More Accurate

In a two-probe method, the measured resistance includes:

contact resistance at the probe–sample interface
lead resistance of the probe wiring
internal resistance of the probe metal
sample resistance

In many cases, the first three factors dominate and distort the measurement.

In the four-probe method:

current flows only through outer probes
the inner probes sense only voltage
the voltmeter draws negligible current

Thus the voltage measured corresponds purely to the sample's intrinsic potential drop, independent of contact and wire resistances.

Real samples are not always semi-infinite. For thin films or small samples, geometric correction factors are applied to compensate for thickness, finite size, and edge effects. However, the core principle and derivation remain the same.

The four-probe method allows high-accuracy measurement of resistivity by isolating voltage measurement from current injection. The expression

$$\rho = 2\pi s \frac{\Delta V}{I}$$

provides a direct way to calculate resistivity from measurable quantities. Once resistivity is known, conductivity is simply its reciprocal. This method is widely used in semiconductor physics, thin-film analysis, and material characterization.

5.9 Hall coefficient

The Hall coefficient is a fundamental property of conducting and semiconducting materials that describes how charge carriers respond to a magnetic field. When a current flows through a material and a magnetic field is applied perpendicular to that current, the charge carriers experience a sideways magnetic force. This force causes them to accumulate on one side of the material, producing a measurable voltage called the Hall voltage.

The Hall coefficient quantifies this effect and is extremely useful for identifying whether the charge carriers are electrons or holes, and for determining the carrier concentration inside the material.

Force Balance and Hall Field

Consider a material carrying current along the x-direction. A magnetic field is applied along the z-direction. The charge carriers (electrons or holes) move with drift velocity and experience the magnetic Lorentz force.

The magnetic force acting on a charge q is:

$$F_B = qv_dB$$

As charges accumulate on one side, they create an electric field across the sample. The electric force acting against the magnetic force is:

$$F_E = qE_H$$

At equilibrium, these two forces balance:

$$qE_H = qv_dB$$

$$E_H = v_dB$$

Drift Velocity and Current Density

The drift velocity relates to the current density:

$$J = nqv_d$$

$$v_d = \frac{J}{nq}$$

Substitute this into the expression for the Hall electric field:

$$E_H = \frac{J}{nq}B$$

Definition of Hall Coefficient

The Hall coefficient $R_H$is defined as:

$$R_H = \frac{E_H}{JB}$$

Substituting the expression for E_H:

$$R_H = \frac{1}{nq}$$

This is the **fundamental formula** for the Hall coefficient.

Significance of the Hall Coefficient

The sign of $R_H$tells us whether the charge carriers are:

positive (holes) → $R_H > 0$

negative (electrons) → $R_H < 0$

The magnitude of $R_H$is inversely proportional to the carrier concentration. A larger Hall coefficient means fewer charge carriers, while a smaller value means higher carrier density.

Hall Voltage

If the sample width is w, the Hall voltage is:

$$V_H = E_H w = \frac{IB}{nqt}$$

where t is the sample thickness.

This formula is used in experiments to precisely determine the carrier concentration in semiconductors.

The Hall coefficient is a key parameter for understanding the electrical behavior of materials. It provides information about:

the type of charge carriers

the concentration of carriers

mobility when combined with conductivity

Because the Hall effect depends directly on charge dynamics, it is one of the most powerful tools in material characterization, especially in semiconductor physics.

5.10 Superconductivity

Superconductivity is a remarkable physical phenomenon in which a material loses all electrical resistance when cooled below a specific temperature known as the critical temperature. In the superconducting state, an electric current can flow indefinitely without any energy loss, making the resistivity drop abruptly to zero. This transition is sharp and well-defined, resembling a phase change in the material.

Superconductors also exhibit perfect diamagnetism, known as the Meissner effect. When the material becomes superconducting, it expels all magnetic field lines from its interior. This behavior is not simply a consequence of zero resistance; instead, it is an intrinsic feature of the superconducting state and demonstrates that superconductivity is a unique quantum phase of matter.

Origin of Superconductivity: Cooper Pair Formation

In normal conductors, electrons interact strongly with lattice vibrations and impurities, causing scattering and generating electrical resistance. However, at very low temperatures, certain materials undergo a cooperative quantum process. Electrons of opposite spin and opposite momentum pair up to form bound states known as Cooper pairs.

These Cooper pairs behave as composite bosons and condense into a single macroscopic quantum state. Because they move in a coordinated manner, they do not scatter like individual electrons. As a result, they flow through the lattice without resistance. The formation of the Cooper pair condensate gives the material long-range quantum order, making superconductivity a fundamentally quantum mechanical phenomenon.

Energy Gap in the Superconducting State

When a material becomes superconducting, an energy gap forms at the Fermi surface. This energy gap corresponds to the binding energy of the Cooper pairs. To break a Cooper pair and return the electrons to a normal conducting state, energy must be supplied. This gap protects the superconducting state from small thermal vibrations and impurity scattering.

The size of the energy gap depends on the material and decreases as the temperature approaches the critical temperature. At the critical temperature, the gap closes and the superconducting state disappears.

Type I and Type II Superconductors

Superconductors are classified into Type I and Type II based on their magnetic behavior.

Type I superconductors, usually pure elemental metals, completely expel magnetic fields until a critical magnetic field value is reached. When this limit is crossed, superconductivity breaks down abruptly.

Type II superconductors, which include alloys and high-temperature superconductors, demonstrate two critical magnetic fields. Between these two fields, magnetic flux enters the material in quantized vortices, while the rest of the material remains superconducting. This allows Type II superconductors to operate under much stronger magnetic fields than Type I, making them useful in applications like MRI machines and particle accelerators.

Loss of Resistance and Persistent Currents

One of the most striking consequences of superconductivity is the existence of persistent currents. Once a current is established in a superconducting loop, it continues without any decay. Experiments have observed persistent currents lasting for years with no measurable reduction. This confirms that superconductors truly possess zero electrical resistance, not just extremely low resistance.

Applications of Superconductivity

Because of their unique properties, superconductors play a vital role in technology. They are used in powerful electromagnets for MRI scanners, magnetic levitation trains, particle accelerators, and quantum computing circuits. In the future, they offer potential for lossless power transmission and ultra-efficient electronic devices.

Superconductivity is a quantum state characterized by zero electrical resistance and complete magnetic field expulsion. It arises from the formation of Cooper pairs and the emergence of a macroscopic quantum phase. The phenomenon has profound theoretical significance and wide-ranging technological applications.

5.11 General properties of superconducting materials

Superconductors are special materials that, below a certain critical temperature, exhibit a complete disappearance of electrical resistance and the expulsion of magnetic fields. These two fundamental features make them unique compared to ordinary conductors. Understanding the general properties of superconductors is essential in exploring their behaviour, applications, and theoretical significance.

Zero Electrical Resistance

One of the most remarkable properties of superconductors is that their electrical resistivity drops abruptly to zero when cooled below the critical temperature (Tc). In this state, electrons flow effortlessly without energy loss. A persistent current set up in a superconducting loop can flow for years without attenuation. This phenomenon indicates the formation of a coherent quantum state in which electrons move in pairs (Cooper pairs), minimizing scattering.

Meissner Effect

Superconductors exhibit perfect diamagnetism below Tc, completely expelling magnetic fields from their interior. This effect, known as the Meissner Effect, distinguishes true superconductivity from mere perfect conductivity. The applied magnetic field is pushed out of the material, and the magnetic induction inside becomes zero. This behaviour reflects the establishment of a new thermodynamic phase.

Critical Magnetic Field (Hc)

Superconductivity can be destroyed by applying an external magnetic field greater than a critical value. This field is called the critical magnetic field (Hc). Below Tc, the value of Hc decreases with increase in temperature. For Type I superconductors, superconductivity is lost suddenly at Hc, whereas Type II superconductors transition through mixed states before losing superconductivity fully.

Critical Temperature (Tc)

Every superconducting material has a characteristic temperature below which it becomes superconducting. This is known as the critical temperature. The discovery of materials with higher Tc values, particularly high-temperature ceramic superconductors (cuprates), has been one of the major breakthroughs in condensed matter physics.

Critical Current Density (Jc)

There is a maximum current that a superconducting material can carry without losing its superconducting state. This limit is known as the critical current density, Jc. When the current exceeds this value, the magnetic field produced by the current can break the Cooper pairs, causing a transition back to the normal state.

Type I and Type II Superconductivity

Superconductors are broadly classified into two types based on their response to magnetic fields:

1. **Type I Superconductors**

   These show a sharp transition into the superconducting state and exhibit complete

Meissner effect until Hc is reached. They are usually pure elemental metals like Pb, Hg, and Al.

2. **Type II Superconductors**

   These materials enter a mixed or vortex state between lower critical field (Hc1) and upper critical field (Hc2). They can withstand very high magnetic fields and are used in magnets for MRI machines and particle accelerators. High-Tc cuprates also belong to this group.

Isotope Effect

The critical temperature of superconductors varies with the isotopic mass of the material. This is known as the isotope effect and provides strong evidence that lattice vibrations (phonons) play a key role in superconductivity. Tc is inversely proportional to the square root of the isotopic mass.

Energy Gap

When a material becomes superconducting, an energy gap opens at the Fermi level. This energy gap represents the binding energy of Cooper pairs. It is a direct consequence of the formation of a correlated quantum state and explains why scattering processes are suppressed at low temperatures.

Perfect Diamagnetism

Due to the Meissner effect, superconductors behave as ideal diamagnets. They completely oppose the applied magnetic field and show susceptibility $\chi = -1$. This behaviour is unique and is not observed even in perfect conductors.

Flux Quantization

In a superconducting ring, the magnetic flux is quantized and can exist only in discrete units of flux quantum ($\Phi_0 = h/2e$). This arises from the coherence of the superconducting wave function and is a direct signature of macroscopic quantum behaviour.

5.12 Critical temperature

The **critical temperature**, denoted as **Tc**, is the specific temperature below which a material enters the superconducting state. At this temperature, the electrical resistivity of the material drops abruptly to **zero**, and the material begins to exhibit the **Meissner effect**, completely expelling magnetic fields from its interior.

The transition at Tc marks a fundamental change in the physical state of the material. Above Tc, the material behaves like a normal conductor, with finite resistivity and partial magnetic penetration. Below Tc, the electrons form **Cooper pairs**, enabling frictionless motion through the crystal lattice.

Temperature Dependence of Superconductivity

When the temperature is gradually lowered toward Tc, the resistivity decreases smoothly, but at Tc it falls discontinuously to zero. This behaviour confirms that superconductivity is a **phase transition**, similar to the transition between liquid and solid states.

The value of Tc is a characteristic property of each material. For example, elemental superconductors like mercury have a low Tc (around 4 K), whereas high-temperature ceramic superconductors show much higher Tc values (above 90 K).

Relation with Critical Magnetic Field

The critical magnetic field also depends on temperature and becomes zero when the temperature reaches Tc. This relation is given by:

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

This shows that superconductivity weakens as the temperature approaches Tc.

Importance of Tc in Applications

A higher Tc makes it easier to cool the material into the superconducting state, reducing cost and enabling practical applications like magnetic levitation, MRI magnets, superconducting power cables, and particle accelerators.

5.13 Critical magnetic field

The **critical magnetic field**, denoted by **H_c**, is the maximum magnetic field that a superconductor can withstand while remaining in the superconducting state. When an external magnetic field stronger than Hc is applied, the material loses its superconducting properties and returns to the normal conducting state.

This phenomenon arises because the superconducting state is characterized by the formation of **Cooper pairs**, which move without resistance. A sufficiently strong magnetic field breaks these pairs, destroying superconductivity.

Temperature Dependence of Critical Magnetic Field

When a material becomes superconducting below its critical temperature T_c, it exhibits perfect diamagnetism — meaning it expels magnetic fields from its interior. This expulsion of magnetic flux occurs up to a certain strength of applied magnetic field known as the **critical magnetic field** H_c.

The critical field varies with temperature and falls to zero at the critical temperature. To derive the theoretical relationship between $H_c$and T, we start with the **thermodynamic free energy** of the superconducting state relative to the normal state.

Gibbs Free Energy Difference

Consider a superconductor in an external magnetic field. Let the Gibbs free energy per unit volume of the normal (non-superconducting) state be G_n, and that of the superconducting state be G_s. At absolute zero and zero magnetic field, the superconducting state has lower free energy than the normal state. As temperature increases or as the magnetic field increases, this difference reduces.

When a magnetic field $H$is applied, the magnetic energy added to the superconductor per unit volume is:

$$\frac{H^2}{2\mu_0}$$

where $\mu_0$is the permeability of free space. The presence of this energy increases the free energy of the superconducting state.

At the **critical field** $H_c(T)$, the free energies of the superconducting and normal states become equal:

$$G_s + \frac{H_c^2(T)}{2\mu_0} = G_n$$

Rearranging, the free energy difference becomes:

$$G_n - G_s = \frac{H_c^2(T)}{2\mu_0}$$

This means the magnetic field needed to suppress superconductivity at temperature $T$is directly related to the energy difference between the normal and superconducting states at that temperature.

Temperature Dependence of Free Energy Difference

The free energy difference between the normal and superconducting states changes with temperature. At absolute zero ($T = 0$), this difference is maximum and corresponds to the maximum critical field $H_c(0)$:

$$G_n - G_s \mid_{T=0} = \frac{H_c^2(0)}{2\mu_0}$$

As temperature increases towards T_c, the superconducting order weakens, and so does the free energy difference. Close to T_c, the free energy difference falls approximately as the square of the reduced temperature:

$$G_n - G_s \propto \left(1 - \frac{T}{T_c}\right)^2$$

This squared dependence arises from thermodynamic considerations of the superconducting order parameter in mean-field theories of phase transitions.

Substituting this behaviour into the earlier relation gives:

$$\frac{H_c^2(T)}{2\mu_0} = \frac{H_c^2(0)}{2\mu_0}\left(1 - \frac{T}{T_c}\right)^2$$

Cancelling the factor $\frac{1}{2\mu_0}$from both sides:

$$H_c^2(T) = H_c^2(0)\left(1 - \frac{T}{T_c}\right)^2$$

Taking the square root of both sides:

$$H_c(T) = H_c(0)\left(1 - \frac{T}{T_c}\right)$$

Final Temperature Dependence Relation

The usual simplified form of the critical field's temperature dependence is written as:

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

This equation shows that the critical magnetic field decreases as temperature increases, and it becomes zero when $T = T_c$. The relation reflects the fact that the superconducting state is strongest at low temperatures and disappears at the critical temperature.

Physical Interpretation

At temperatures close to absolute zero, the superconducting state is robust, and the material tolerates higher magnetic fields before losing superconductivity. As temperature rises toward T_c, thermal agitation weakens the collective Cooper pair state, reducing the ability of the material to expel magnetic fields. Therefore, a lower magnetic field is sufficient to destroy superconductivity.

This temperature dependence of the critical magnetic field is fundamental to understanding superconducting devices and their operating limits under varying temperatures and magnetic environments.

Physical Meaning

Below Hc, the material exhibits the **Meissner effect**, completely expelling magnetic flux from its interior. This perfect diamagnetism is a defining property of superconductors. However, once the applied field crosses Hc, magnetic flux penetrates through the material, destroying the superconducting state.

Type I and Type II Superconductors

In **Type I superconductors**, there is a single critical field Hc. The material switches abruptly from superconducting to normal state once Hc is exceeded.

In **Type II superconductors**, two critical fields exist:

Lower critical field (H_{c1})

Upper critical field (H_{c2})

Between these fields, the material enters a **mixed (vortex) state**, where magnetic flux penetrates in quantized vortices while the material still retains partial superconductivity. Above Hc2, the material becomes fully normal.

Importance of Critical Magnetic Field

Understanding Hc is essential for designing devices such as superconducting magnets, MRI systems, magnetic levitation trains, and particle accelerators. If the applied magnetic field exceeds the permissible limit, superconductivity collapses and the system may lose efficiency or become unstable.

5.14 Meissner effect

The **Meissner Effect** is the fundamental electromagnetic property of superconductors where they **completely expel magnetic fields** from their interior when cooled below the critical temperature. This phenomenon shows that superconductivity is a distinct thermodynamic state, not just perfect electrical conductivity.

In ordinary conductors with zero resistance, a magnetic field would become “frozen in” when cooled in the presence of that field. However, superconductors do more than conduct without resistance—they actively **push magnetic field lines out** as they transition into the superconducting state. This total expulsion of magnetic flux demonstrates that the superconducting state is characterized by a unique electromagnetic order.

Perfect Diamagnetism

The Meissner Effect makes superconductors **perfect diamagnets**. Diamagnetism refers to the tendency of a material to create an induced magnetic field in the opposite direction of an applied field. In superconductors, this induced field exactly cancels the applied magnetic field inside the material. The result is that the magnetic induction within the superconductor becomes zero.

Because magnetic flux lines are excluded, a magnet placed near a superconductor can levitate above it—a dramatic and often-shown demonstration of the Meissner Effect. This levitation occurs because the superconductor creates currents on its surface that generate a magnetic field opposing the applied one, effectively locking the magnet in place.

Difference from Perfect Conductivity

Zero electrical resistance alone does not guarantee the Meissner Effect. A hypothetical perfect conductor with zero resistance could maintain an existing magnetic field within it if the field was present while it cooled. A superconductor, however, always expels magnetic fields upon entering the superconducting state, regardless of history. This behavior confirms that superconductivity is a **thermodynamic phase with distinct electromagnetic properties**, not merely an absence of resistive loss.

Magnetic Field Behavior at the Transition

When a material transitions into the superconducting state, the magnetic field lines that previously entered the material are expelled or pushed out toward the surface. This results in:

• Magnetic induction inside the superconductor becoming zero

• Screening currents developing on the surface to oppose applied fields

• A characteristic field penetration depth where the field decays rapidly near the surface

This penetration depth is finite and reflects how the superconducting electrons screen the field.

Significance of the Meissner Effect

The Meissner Effect is central to all superconducting behavior. It:

• Distinguishes superconductors from normal conductors

• Explains magnetic levitation and flux expulsion phenomena

• Underpins the perfect diamagnetism and electromagnetic stability of superconductors

• Demonstrates that superconductivity is a collective quantum state

This effect is observed in both Type I and Type II superconductors, though in Type II, magnetic flux can penetrate in quantized vortices when the applied field exceeds the lower critical field.

The Meissner Effect describes how superconductors expel magnetic fields when cooled below the critical temperature, leading to perfect diamagnetism. It shows that superconductivity is not simply zero resistance but a **distinct electromagnetic phase**, with unique field-expulsion behavior that enables levitation and flux quantization.

5.15 Isotope effect

The **isotope effect** is an important experimental observation in superconductivity that provides deep insight into the mechanism behind the phenomenon. It refers to the change in the **critical temperature** of a superconductor when the isotopic mass of its constituent atoms is altered.

Isotopes are atoms of the same element that have the same number of protons but different numbers of neutrons. Because their chemical properties are nearly identical, changes in isotopic mass do not affect the electronic structure directly. However, heavier isotopes have **lower vibrational frequencies** (phonons) because the atoms vibrate more sluggishly compared to lighter isotopes.

Connection to Superconductivity

Superconductivity depends critically on interactions between electrons and lattice vibrations (phonons). In conventional superconductors, electrons form paired states known as **Cooper pairs** through mediation by phonons. The strength of this interaction influences the critical temperature T_c, below which superconductivity emerges.

The isotope effect shows that when atoms in a superconducting material are replaced with heavier isotopes, the critical temperature decreases. This implies that lattice vibrations play a key role in electron pairing: heavier atoms vibrate more slowly (lower phonon frequencies), weakening the electron-phonon coupling and thereby lowering T_c.

The isotope effect in superconductors can be expressed by the relation:

$$T_c \propto \frac{1}{\sqrt{M}}$$

Here, M is the isotopic mass of the lattice atoms. This equation predicts that the critical temperature is inversely proportional to the square root of the isotopic mass. Replacing atoms with heavier isotopes causes a measurable reduction in T_c, providing direct evidence that **phonons influence superconductivity**.

The discovery of the isotope effect in the 1950s was a pivotal moment in the development of the microscopic theory of superconductivity. It provided experimental support for the idea that lattice vibrations are intimately connected to pairing mechanisms — a cornerstone of the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity.

The isotope effect reveals that the **critical temperature of a superconductor depends on the isotopic mass** of its constituent atoms. Lighter isotopes correspond to higher phonon frequencies and stronger electron-phonon interactions, which elevate the superconducting transition temperature. This effect underscores the essential role of lattice vibrations in the superconducting state.

5.16 Type-I & Type-II Superconductors

Superconductors are broadly classified into **Type-I** and **Type-II** based on how they respond to external magnetic fields. This classification is essential for understanding their magnetic behavior, practical applications, and the physics underlying the Meissner effect in different materials.

Type-I superconductors are typically pure elemental metals that exhibit an abrupt transition from the superconducting state to the normal state when the applied magnetic field exceeds a certain value. Type-II superconductors, on the other hand, are usually alloys or complex materials that display two distinct critical fields and a mixed state where magnetic flux partially penetrates the material.

Type-I Superconductors

Type-I superconductors are known as **soft superconductors**. They exhibit a *complete* Meissner effect — meaning they expel magnetic flux entirely as long as the applied magnetic field is below the critical field. When the applied magnetic field becomes greater than the critical field, superconductivity vanishes suddenly, and the material reverts to the normal state.

These materials include pure metals such as lead, mercury, tin, and aluminum. Their critical fields are relatively low, typically below 0.1 tesla. Because of this low limit, Type-I superconductors are not suitable for applications requiring strong magnetic fields. However, they are valuable in fundamental research because their behavior is simpler and more ideal for studying the physics of superconductivity.

Type-II Superconductors

Type-II superconductors are known as **hard superconductors**. They behave differently by possessing **two critical magnetic fields**. Below the lower critical field, they exhibit perfect flux expulsion, similar to Type-I superconductors. Between the lower and upper critical fields, the material enters a unique state called the **mixed state**, where magnetic flux penetrates partially in the form of quantized flux lines or vortices. These vortices contain normal-conducting cores surrounded by circulating supercurrents.

This mixed state allows Type-II superconductors to operate under much stronger magnetic fields — often several tesla — making them suitable for real-world applications such as MRI machines, particle accelerators, magnetic levitation systems, and high-power electromagnets. Materials such as niobium-titanium (Nb-Ti) and niobium-tin (Nb_3Sn) are commonly used Type-II superconductors.

Mixed State and Flux Vortices

A defining feature of Type-II superconductors is the existence of quantized flux vortices in the mixed state. These vortices arrange themselves in a regular pattern known as the Abrikosov lattice. The presence of these vortices allows magnetic field penetration while still maintaining superconductivity outside the vortex cores.

The movement of vortices can lead to energy dissipation, but pinning centers within the material can lock the vortices in place, enhancing current-carrying capability and making Type-II superconductors highly practical.

Type-I superconductors exhibit an abrupt transition from superconducting to normal state with a single critical field and demonstrate complete flux expulsion. They are typically pure metals and are limited to low magnetic fields. Type-II superconductors possess two critical fields and a mixed state where magnetic flux lines penetrate as vortices. This class of superconductors operates under high magnetic fields and is widely used in technological applications due to its superior performance in magnetic environments.

5.17 London's equation and penetration depth

The London brothers introduced two phenomenological equations to describe the electromagnetic behavior of superconductors. These equations successfully explain the Meissner effect and lead to the concept of penetration depth. They treat superconducting electrons as a rigid, lossless fluid that responds collectively to electromagnetic fields.

In the superconducting state, electrons move without resistance and generate screening currents that prevent magnetic fields from entering the material. The London equations connect the supercurrent density to the electromagnetic fields present inside the superconductor.

First London Equation

The first London equation states that the rate of change of supercurrent density is proportional to the electric field inside the superconductor:

$$\frac{d\mathbf{J_s}}{dt} = \frac{n_s e^2}{m}\mathbf{E}$$

Here,

$\mathbf{J_s}$ is the supercurrent density,

n_s is the density of superconducting electrons,

e is electron charge,

m is electron mass.

This equation indicates that an electric field cannot exist inside a superconductor for long; the supercurrent quickly adjusts itself to cancel the electric field. This explains infinite conductivity.

Second London Equation

The second London equation describes the relationship between supercurrent density and magnetic field:

$$\nabla \times \mathbf{J_s} = -\frac{n_s e^2}{m}\mathbf{B}$$

This equation leads directly to the **exponential decay of magnetic field** inside a superconductor — the essence of the Meissner effect.

Derivation of Penetration Depth

To find how the magnetic field decays within a superconductor, combine the second London equation with Maxwell's equation:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J_s}$$

Taking curl on both sides of the London equation:

$$\nabla \times (\nabla \times \mathbf{J_s}) = -\frac{n_s e^2}{m}(\nabla \times \mathbf{B})$$

Using vector identity:

$$\nabla \times (\nabla \times \mathbf{J_s}) = \nabla(\nabla \cdot \mathbf{J_s}) - \nabla^2 \mathbf{J_s}$$

Since the supercurrent is divergence-free:

$$\nabla \cdot \mathbf{J_s} = 0$$

So:

$$-\nabla^2 \mathbf{J_s} = -\frac{n_s e^2}{m}(\mu_0 \mathbf{J_s})$$

Thus

$$\nabla^2 \mathbf{J_s} = \frac{1}{\lambda_L^2}\mathbf{J_s}$$

Where the London penetration depth is:

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

Magnetic Field Decay

Using Maxwell's relation:

$$\nabla^2 \mathbf{B} = \frac{1}{\lambda_L^2}\mathbf{B}$$

The solution for a magnetic field entering a flat superconducting surface at $x = 0$is:

$$B(x) = B_0 e^{-x/\lambda_L}$$

This shows the magnetic field falls exponentially inside the superconductor.

Physical Meaning of Penetration Depth

The penetration depth represents the distance over which the magnetic field reduces to $1/e$of its surface value. It arises because supercurrents flowing near the surface oppose the applied magnetic field. These currents are confined to a thin layer only a few hundred nanometers thick, depending on the material.

A smaller penetration depth means stronger superconductivity because the material expels magnetic fields more effectively. Type-I superconductors usually have small penetration depths, while Type-II superconductors have larger values.

London equations describe how superconducting currents respond to electric and magnetic fields. Their combination shows that magnetic fields are expelled from the interior of a superconductor, except for a thin surface region where they decay exponentially. This characteristic distance is the London penetration depth, a fundamental parameter that determines magnetic behaviour of superconductors.